In this Booklet:

1. Hardest Past paper questions between 2006-2013 (+ june 2015)
2. Hardest mixed exercise questions from the text book
3. Hardest Solomon paper questions

Answers can all be found individually on Physics and Maths tutor, including mixed exercises solutions in the solution bank section.

But complete answer booklet for this can be found on StudentGrounds.

StudentGrounds

*Compared to C1, C2 past papers were generally quite repetitive, however the Solomon papers were **<u>much</u>** harder.

Hardest Past Paper Questions

June 2015

7. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2(11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

June 2015

8. (i) Solve, for $0 \leqslant \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π.

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \qquad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k.

(3)

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$

(3)

June 2015

9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

June 2013

6.

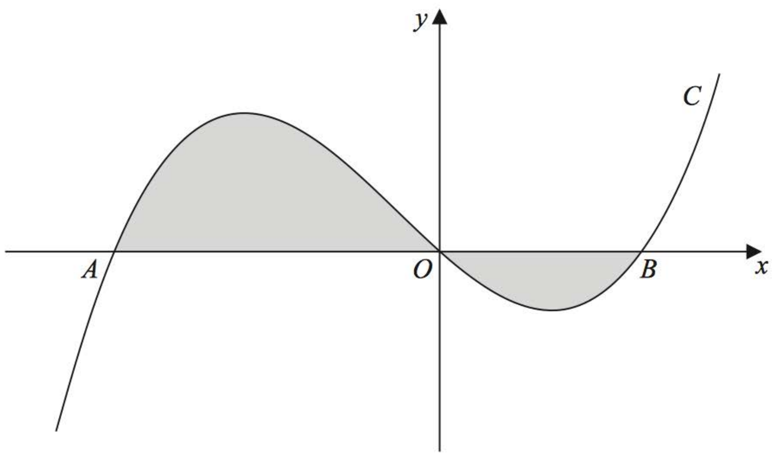

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

10.

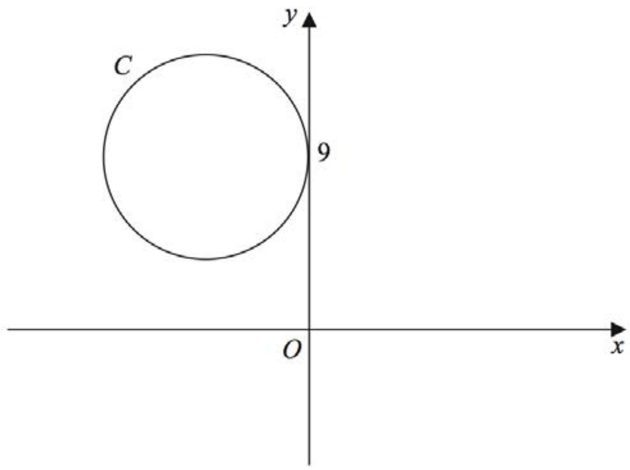

Figure 4

The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in Figure 4.

(a) Write down an equation for the circle C, that is shown in Figure 4.

(3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T.

(b) Find the length of PT.

(3)

June 2013 R

6. Given that $\log_3 x = a$, find in terms of a,

(a) $\log_3 (9x)$

(2)

(b) $\log_3 \left(\dfrac{x^5}{81}\right)$

(3)

giving each answer in its simplest form.

(c) Solve, for x,

$$\log_3 (9x) + \log_3 \left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures.

(4)

Jan 2013

3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

(1)

(b) Find the first year in which the yearly predicted profit exceeds £200 000

(5)

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

5. The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M.

(a) Find

(i) the coordinates of the point M,

(ii) the radius of the circle C.

(5)

N is the point with coordinates (25, 32).

(b) Find the length of the line MN.

(2)

The tangent to C at a point P on the circle passes through point N.

(c) Find the length of the line NP.

(2)

June 2012

6. (a) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leqslant x \leqslant 180°$,

$$\tan 2x = 5 \sin 2x$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

January 2012

3. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

4. Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3(28x - 9)$$

(3)

9. (i) Find the solutions of the equation $\sin(3x - 15°) = \frac{1}{2}$, for which $0 \leqslant x \leqslant 180°$

(6)

(ii)

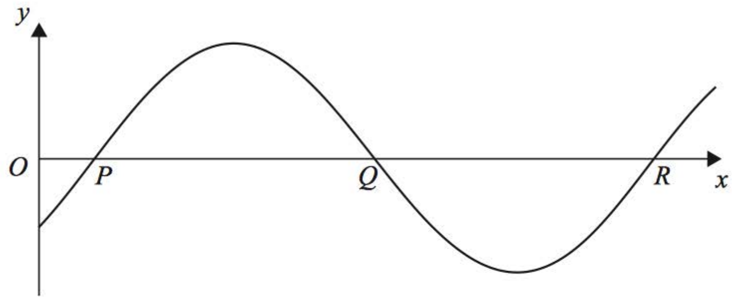

Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0,\ 0 < b < \pi$$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{3\pi}{5}, 0\right)$ and $\left(\frac{11\pi}{10}, 0\right)$ respectively, find the values of a and b.

(4)

June 2011:

5.

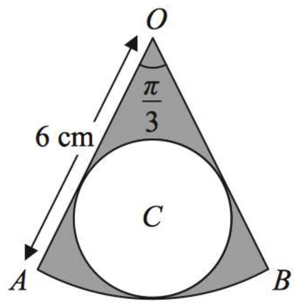

Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector *OAB* of a circle centre *O*, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle *C*, inside the sector, touches the two straight edges, *OA* and *OB*, and the arc *AB* as shown.

Find

(a) the area of the sector *OAB*,

(2)

(b) the radius of the circle *C*.

(3)

The region outside the circle *C* and inside the sector *OAB* is shown shaded in Figure 1.

(c) Find the area of the shaded region.

(2)

Jan 2011

5. Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

June 2010

10. The circle C has centre $A(2, 1)$ and passes through the point $B(10, 7)$.

(a) Find an equation for C.

(4)

The line l_1 is the tangent to C at the point B.

(b) Find an equation for l_1.

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB.

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)

Jan 2010

6. A car was purchased for £18 000 on 1st January.
On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.
(1)

The value of the car falls below £1000 for the first time n years after it was purchased.

(b) Find the value of n.
(3)

An insurance company has a scheme to cover the maintenance of the car.
The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.
(2)

(d) Find the total cost of the insurance scheme for the first 15 years.
(3)

8.

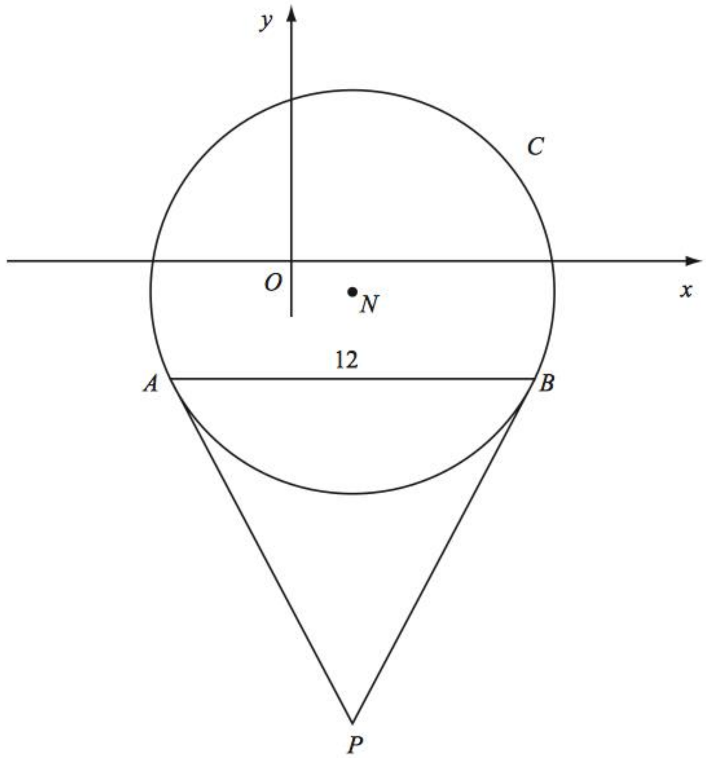

Figure 3

Figure 3 shows a sketch of the circle C with centre N and equation

$$(x-2)^2+(y+1)^2=\frac{169}{4}$$

(a) Write down the coordinates of N.

(2)

(b) Find the radius of C.

(1)

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12 units as shown in Figure 3.

(c) Find the coordinates of A and the coordinates of B.

(5)

(d) Show that angle $ANB = 134.8°$, to the nearest 0.1 of a degree.

(2)

The tangents to C at the points A and B meet at the point P.

(e) Find the length AP, giving your answer to 3 significant figures.

(2)

June 2009

8. (a) Find the value of y such that

$$\log_2 y = -3$$

(2)

(b) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)

Jan 2009

5.

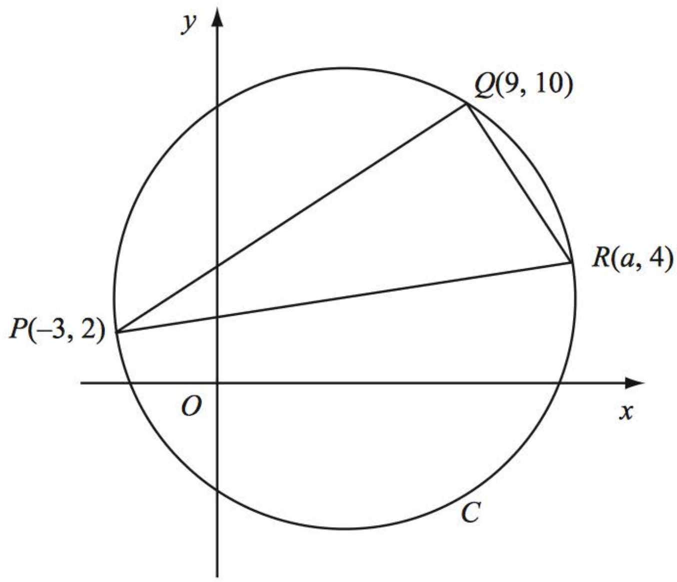

Figure 2

The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in Figure 2.
Given that PR is a diameter of C,

(a) show that $a = 13$,

(3)

(b) find an equation for C.

(5)

June 2008

6. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(a) the 20th term of the series, to 3 decimal places,

(2)

(b) the sum to infinity of the series.

(2)

Given that the sum to k terms of the series is greater than 24.95,

(c) show that $k > \dfrac{\log 0.002}{\log 0.8}$,

(4)

(d) find the smallest possible value of k.

(1)

June 2007

8. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \frac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

Jan 2007

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1-9x .$$

(2)

June 2006

10.

Figure 3

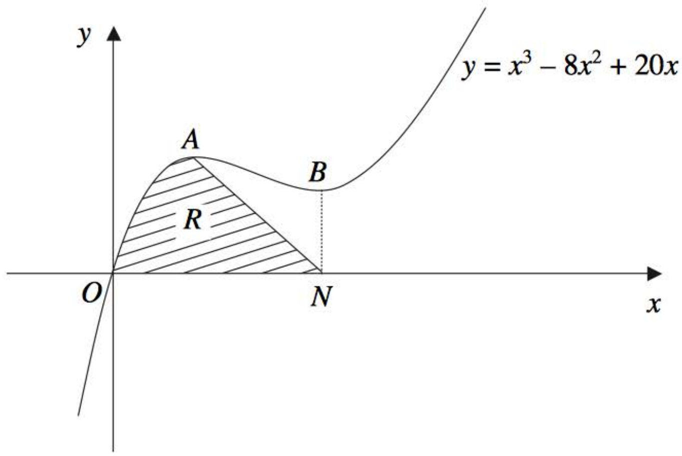

Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$.

(3)

(d) Hence calculate the exact area of R.

(5)

Hardest Mixed Exercises Questions

Mixed excersises text book:
Solutions: http://www.physicsandmathstutor.com/a-level-maths-papers/c2-solutionbank/

Exponentials and logarithms
Exercise G, Question 10

Question:

Prove that if $a^x = b^y = (ab)^{xy}$, then $x + y = 1$. **[E]**

Exponentials and logarithms

Exercise G, Question 11

Question:

(a) Show that $\log_4 3 = \log_2 \sqrt{3}$.

(b) Hence or otherwise solve the simultaneous equations:
$2 \log_2 y = \log_4 3 + \log_2 x$,
$3^y = 9^x$,
given that x and y are positive. **[E]**

Algebra and functions
Exercise A, Question 29

Question:

Solve the simultaneous equations

$4 \log_9 x + 4 \log_3 y = 9$

$6 \log_3 x + 6 \log_{27} y = 7$

Algebra and functions
Exercise A, Question 31

Question:

The circle C has equation $x^2 + y^2 - 10x + 4y + 20 = 0$.
Find the length of the tangent to C from the point $(-4, 4)$.

The binomial expansion

Exercise E, Question 8

Question:

(a) Expand $(1 + 2x)^{12}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

(b) By substituting a suitable value for x, which must be stated, into your answer to part (a), calculate an approximate value of $(1.02)^{12}$.

(c) Use your calculator, writing down all the digits in your display, to find a more exact value of $(1.02)^{12}$.

(d) Calculate, to 3 significant figures, the percentage error of the approximation found in part (b).

The binomial expansion
Exercise E, Question 6

Question:

The coefficient of x^2 in the binomial expansion of $\left(1 + \frac{x}{2}\right)^n$, where n is a positive integer, is 7.

(a) Find the value of n.

(b) Using the value of n found in part (a), find the coefficient of x^4.

The binomial expansion

Exercise E, Question 10

Question:

In the binomial expansion of $(2k + x)^n$, where k is a constant and n is a positive integer, the coefficient of x^2 is equal to the coefficient of x^3.

(a) Prove that $n = 6k + 2$.

(b) Given also that $k = \frac{2}{3}$, expand $(2k + x)^n$ in ascending powers of x up to and including the term in x^3, giving each coefficient as an exact fraction in its simplest form.

Hardest Solomon Paper Questions

Solomon worksheet C circles

13 The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.

a Show that l passes through the centre of C. **(3)**

b Find an equation for each tangent to C that is parallel to l. **(6)**

Solomon A

9.

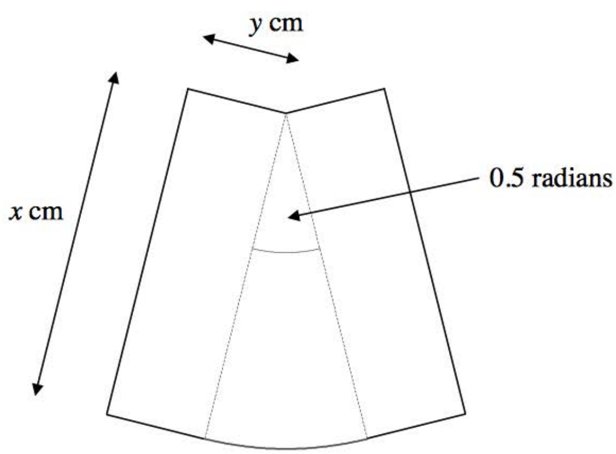

Figure 3

Figure 3 shows a design consisting of two rectangles measuring x cm by y cm joined to a circular sector of radius x cm and angle 0.5 radians.

Given that the area of the design is 50 cm^2,

(a) show that the perimeter, P cm, of the design is given by

$$P = 2x + \frac{100}{x}.$$ **(5)**

(b) Find the value of x for which P is a minimum. **(4)**

(c) Show that P is a minimum for this value of x. **(2)**

(d) Find the minimum value of P in the form $k\sqrt{2}$. **(2)**

Solomon B

3. For the binomial expansion in ascending powers of x of $(1 + \frac{1}{4}x)^n$, where n is an integer and $n \geq 2$,

(a) find and simplify the first three terms, **(3)**

(b) find the value of n for which the coefficient of x is equal to the coefficient of x^2. **(3)**

5. The circle C has centre $(-1, 6)$ and radius $2\sqrt{5}$.

(a) Find an equation for C. **(2)**

The line $y = 3x - 1$ intersects C at the points A and B.

(b) Find the x-coordinates of A and B. **(4)**

(c) Show that $AB = 2\sqrt{10}$. **(3)**

7. A student completes a mathematics course and begins to work through past exam papers. He completes the first paper in 2 hours and the second in 1 hour 54 minutes.

Assuming that the times he takes to complete successive papers form a geometric sequence,

(a) find, to the nearest minute, how long he will take to complete the fifth paper, **(3)**

(b) show that the total time he takes to complete the first eight papers is approximately 13 hours 28 minutes, **(3)**

(c) find the least number of papers he must work through if he is to complete a paper in less than one hour. **(4)**

Solomon C

4. *(a)* *(i)* Sketch the curve $y = \sin(x - 30)°$ for x in the interval $-180 \leq x \leq 180$.

(ii) Write down the coordinates of the turning points of the curve in this interval. **(4)**

(b) Find all values of x in the interval $-180 \leq x \leq 180$ for which

$$\sin(x - 30)° = 0.35,$$

giving your answers to 1 decimal place. **(4)**

5. *(a)* Evaluate

$$\log_3 27 - \log_8 4.$$ **(4)**

(b) Solve the equation

$$4^x - 3(2^{x+1}) = 0.$$ **(5)**

7. The points P, Q and R have coordinates $(-5, 2)$, $(-3, 8)$ and $(9, 4)$ respectively.

(a) Show that $\angle PQR = 90°$. **(4)**

Given that P, Q and R all lie on circle C,

(b) find the coordinates of the centre of C, **(3)**

(c) show that the equation of C can be written in the form

$$x^2 + y^2 - 4x - 6y = k,$$

where k is an integer to be found. **(3)**

8.

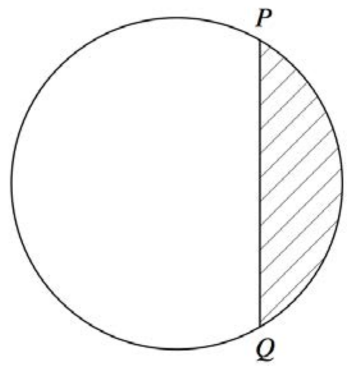

Figure 2

Figure 2 shows a circle of radius 12 cm which passes through the points P and Q.
The chord PQ subtends an angle of 120° at the centre of the circle.

(a) Find the exact length of the major arc PQ. **(2)**

(b) Show that the perimeter of the shaded minor segment is given by $k(2\pi + 3\sqrt{3})$ cm, where k is an integer to be found. **(4)**

(c) Find, to 1 decimal place, the area of the shaded minor segment as a percentage of the area of the circle. **(4)**

Solomon D

5. *(a)* Sketch the curve $y = 5^{x-1}$, showing the coordinates of any points of intersection with the coordinate axes. **(2)**

(b) Find, to 3 significant figures, the x-coordinates of the points where the curve $y = 5^{x-1}$ intersects

(i) the straight line $y = 10$,

(ii) the curve $y = 2^x$. **(6)**

7. *(a)* Prove that the sum of the first n terms of a geometric series with first term a and common ratio r is given by

$$\frac{a(1-r^n)}{1-r}.$$ **(4)**

(b) Evaluate $\sum_{r=1}^{12} (5 \times 2^r)$. **(5)**

9.

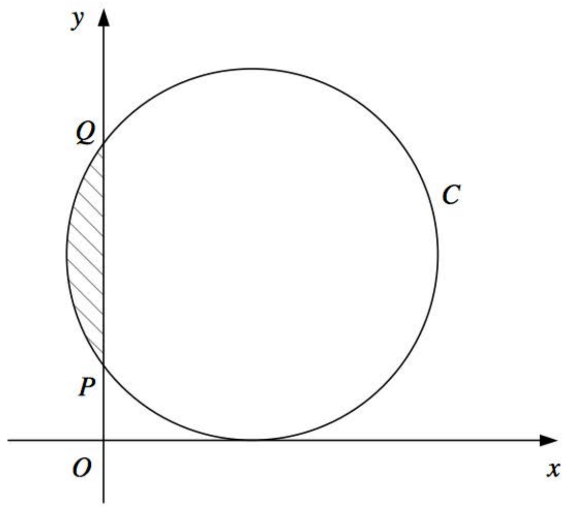

Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 8x - 10y + 16 = 0.$$

(a) Find the coordinates of the centre and the radius of C. **(3)**

C crosses the y-axis at the points P and Q.

(b) Find the coordinates of P and Q. **(3)**

The chord PQ subtends an angle of θ at the centre of C.

(c) Using the cosine rule, show that $\cos\theta = \frac{7}{25}$. **(4)**

(d) Find the area of the shaded minor segment bounded by C and the chord PQ. **(4)**

Solomon E

3. Given that $p = \log_2 3$ and $q = \log_2 5$, find expressions in terms of p and q for

(a) $\log_2 45$, **(3)**

(b) $\log_2 0.3$ **(3)**

5. *(a)* Write down the exact value of $\cos \frac{\pi}{6}$. **(1)**

The finite region R is bounded by the curve $y = \cos^2 x$, where x is measured in radians, the positive coordinate axes and the line $x = \frac{\pi}{3}$.

(b) Use the trapezium rule with three equally-spaced ordinates to estimate the area of R, giving your answer to 3 significant figures. **(5)**

The finite region S is bounded by the curve $y = \sin^2 x$, where x is measured in radians, the positive coordinate axes and the line $x = \frac{\pi}{3}$.

(c) Using your answer to part *(b)*, find an estimate for the area of S. **(3)**

6.

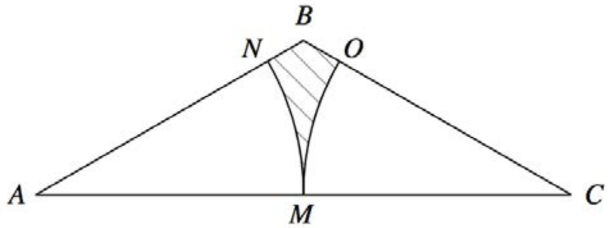

Figure 1

Figure 1 shows triangle *ABC* in which $AC = 8$ cm and $\angle BAC = \angle BCA = 30°$.

(a) Find the area of triangle *ABC* in the form $k\sqrt{3}$. **(5)**

The point *M* is the mid-point of *AC* and the points *N* and *O* lie on *AB* and *BC* such that *MN* and *MO* are arcs of circles with centres *A* and *C* respectively.

(b) Show that the area of the shaded region *BNMO* is $\frac{8}{3}(2\sqrt{3} - \pi)$ cm^2. **(4)**

7. The circle C has the equation

$$x^2 + y^2 + 10x - 8y + k = 0,$$

where k is a constant.

Given that the point with coordinates (–6, 5) lies on C,

(a) find the value of k, **(2)**

(b) find the coordinates of the centre and the radius of C. **(3)**

A straight line which passes through the point A (2, 3) is a tangent to C at the point B.

(c) Find the length AB in the form $k\sqrt{3}$. **(5)**

8. Amy plans to join a savings scheme in which she will pay in £500 at the start of each year.

One scheme that she is considering pays 6% interest on the amount in the account at the end of each year.

For this scheme,

(a) find the amount of interest paid into the account at the end of the second year, **(3)**

(b) show that after interest is paid at the end of the eighth year, the amount in the account will be £5246 to the nearest pound. **(4)**

Another scheme that she is considering pays 0.5% interest on the amount in the account at the end of each month.

(c) Find, to the nearest pound, how much more or less will be in the account at the end of the eighth year under this scheme. **(5)**

9. The polynomial f(x) is given by

$$f(x) = x^3 + kx^2 - 7x - 15,$$

where k is a constant.

When f(x) is divided by (x + 1) the remainder is r.

When f(x) is divided by (x – 3) the remainder is $3r$.

(a) Find the value of k. **(5)**

(b) Find the value of r. **(1)**

(c) Show that (x – 5) is a factor of f(x). **(2)**

(d) Show that there is only one real solution to the equation $f(x) = 0$. **(4)**

Solomon F

4. *(a)* Expand $(2 + y)^6$ in ascending powers of y as far as the term in y^3, simplifying each coefficient. **(4)**

(b) Hence expand $(2 + x - x^2)^6$ in ascending powers of x as far as the term in x^3, simplifying each coefficient. **(3)**

6. *(a)* Given that $y = 3^x$, find expressions in terms of y for

(i) 3^{x+1},

(ii) 3^{2x-1}. **(4)**

(b) Hence, or otherwise, solve the equation

$$3^{x+1} - 3^{2x-1} = 6,$$

giving non-exact answers to 2 decimal places. **(5)**

Solomon G

4. Solve each equation, giving your answers to an appropriate degree of accuracy.

(*a*) $3^{x-2} = 5$ **(3)**

(*b*) $\log_2 (6 - y) = 3 - \log_2 y$ **(4)**

8.

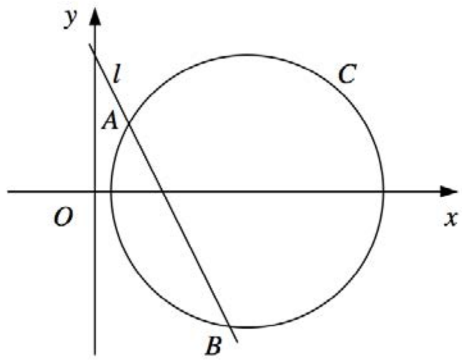

Figure 2

Figure 2 shows the circle C and the straight line l. The centre of C lies on the x-axis and l intersects C at the points A (2, 4) and B (8, −8).

(a) Find the gradient of l. **(2)**

(b) Find the coordinates of the mid-point of AB. **(2)**

(c) Find the coordinates of the centre of C. **(5)**

(d) Show that C has the equation $x^2 + y^2 - 18x + 16 = 0$. **(3)**

9.

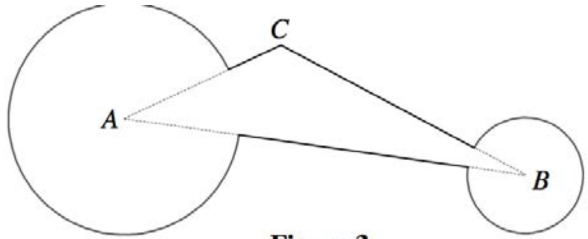

Figure 3

Figure 3 shows a design painted on the wall at a karting track. The sign consists of triangle *ABC* and two circular sectors of radius 2 metres and 1 metre with centres *A* and *B* respectively.

Given that $AB = 7$ m, $AC = 3$ m and $\angle ACB = 2.2$ radians,

(*a*) use the sine rule to find the size of $\angle ABC$ in radians to 3 significant figures, **(3)**

(*b*) show that $\angle BAC = 0.588$ radians to 3 significant figures, **(2)**

(*c*) find the area of triangle *ABC*, **(2)**

(*d*) find the area of the wall covered by the design. **(5)**

Solomon H

5. *(a)* Describe fully a single transformation that maps the graph of $y = 3^x$ onto the graph of $y = (\frac{1}{3})^x$. **(1)**

(b) Sketch on the same diagram the curves $y = (\frac{1}{3})^x$ and $y = 2(3^x)$, showing the coordinates of any points where each curve crosses the coordinate axes. **(3)**

The curves $y = (\frac{1}{3})^x$ and $y = 2(3^x)$ intersect at the point P.

(c) Find the x-coordinate of P to 2 decimal places and show that the y-coordinate of P is $\sqrt{2}$. **(5)**

8. (a) Given that $\sin\theta = 2 - \sqrt{2}$, find the value of $\cos^2\theta$ in the form $a + b\sqrt{2}$ where a and b are integers. **(3)**

(b) Find, in terms of π, all values of x in the interval $0 \leq x < \pi$ for which

$$\cos\left(2x - \frac{\pi}{6}\right) = \frac{1}{2}.$$ **(7)**

9. The second and fifth terms of a geometric series are –48 and 6 respectively.

(a) Find the first term and the common ratio of the series. **(5)**

(b) Find the sum to infinity of the series. **(2)**

(c) Show that the difference between the sum of the first n terms of the series and its sum to infinity is given by 2^{6-n}. **(5)**

Solomon I

7. Given that for small values of x

$$(1 + ax)^n \approx 1 - 24x + 270x^2,$$

where n is an integer and $n > 1$,

(a) show that $n = 16$ and find the value of a, **(7)**

(b) use your value of a and a suitable value of x to estimate the value of $(0.9985)^{16}$, giving your answer to 5 decimal places. **(3)**

8. *(a)* Given that

$$\log_2 (y - 1) = 1 + \log_2 x,$$

show that

$$y = 2x + 1. \qquad \textbf{(3)}$$

(b) Solve the simultaneous equations

$$\log_2 (y - 1) = 1 + \log_2 x$$

$$2 \log_3 y = 2 + \log_3 x \qquad \textbf{(7)}$$

9.

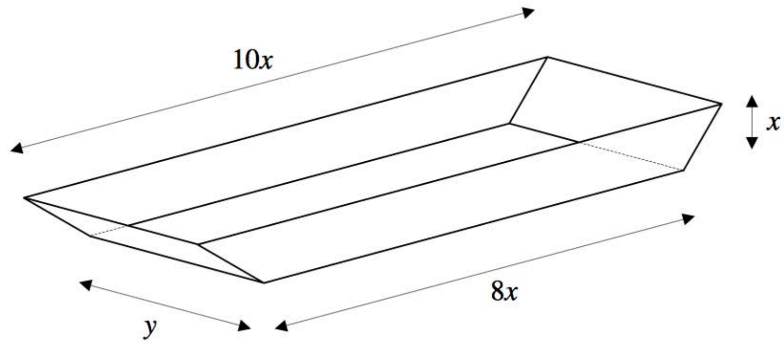

Figure 2

Figure 2 shows a tray made from sheet metal.

The horizontal base is a rectangle measuring $8x$ cm by y cm and the two vertical sides are trapezia of height x cm with parallel edges of length $8x$ cm and $10x$ cm. The remaining two sides are rectangles inclined at 45° to the horizontal.

Given that the capacity of the tray is 900 cm^3,

(a) find an expression for y in terms of x, **(3)**

(b) show that the area of metal used to make the tray, A cm^2, is given by

$$A = 18x^2 + \frac{200(4+\sqrt{2})}{x},$$ **(4)**

(c) find to 3 significant figures, the value of x for which A is stationary, **(4)**

(d) find the minimum value of A and show that it is a minimum. **(3)**

Solomon J

6. $\mathrm{f}(x) = \cos 2x, \quad 0 \le x \le \pi.$

(a) Sketch the curve $y = \mathrm{f}(x)$. **(2)**

(b) Write down the coordinates of any points where the curve $y = \mathrm{f}(x)$ meets the coordinate axes. **(3)**

(c) Solve the equation $\mathrm{f}(x) = 0.5$, giving your answers in terms of π. **(4)**

8. The second and third terms of a geometric series are $\log_3 4$ and $\log_3 16$ respectively.

(a) Find the common ratio of the series. **(3)**

(b) Show that the first term of the series is $\log_3 2$. **(2)**

(c) Find, to 3 significant figures, the sum of the first six terms of the series. **(4)**

StudentGrounds

Solomon K

3. *(a)* Given that $y = \log_2 x$, find expressions in terms of y for

(i) $\log_2 \left(\frac{x}{2}\right)$,

(ii) $\log_2 (\sqrt{x})$. **(4)**

(b) Hence, or otherwise, solve the equation

$$2 \log_2 \left(\frac{x}{2}\right) + \log_2 (\sqrt{x}) = 8.$$ **(3)**

4. $$f(x) = 2 - x - x^3.$$

(a) Show that $f(x)$ is decreasing for all values of x. **(4)**

(b) Verify that the point $(1, 0)$ lies on the curve $y = f(x)$. **(1)**

(c) Find the area of the region bounded by the curve $y = f(x)$ and the coordinate axes. **(4)**

7. *(a)* Find, to 2 decimal places, the values of x in the interval $0 \leq x < 2\pi$ for which

$$\tan\left(x + \frac{\pi}{4}\right) = 3.$$ **(4)**

(b) Find, in terms of π, the values of y in the interval $0 \leq y < 2\pi$ for which

$$2 \sin y = \tan y.$$ **(6)**

9.

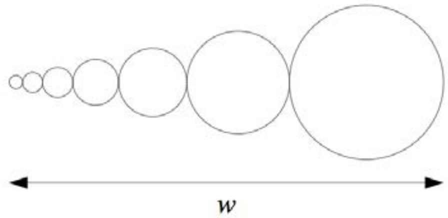

Figure 3

Figure 3 shows part of a design being produced by a computer program.

The program draws a series of circles with each one touching the previous one and such that their centres lie on a horizontal straight line.

The radii of the circles form a geometric sequence with first term 1 mm and second term 1.5 mm. The width of the design is w as shown.

(a) Find the radius of the fourth circle to be drawn. **(2)**

(b) Show that when eight circles have been drawn, $w = 98.5$ mm to 3 significant figures. **(4)**

(c) Find the total area of the design in square centimetres when ten circles have been drawn. **(5)**

Solomon L

3.

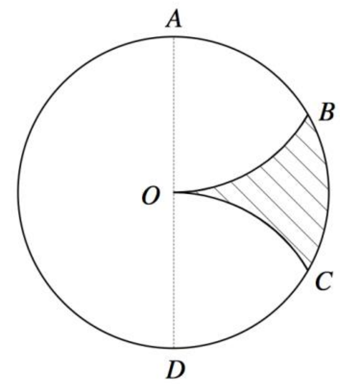

Figure 1

Figure 1 shows a circle of radius r and centre O in which AD is a diameter.

The points B and C lie on the circle such that OB and OC are arcs of circles of radius r with centres A and D respectively.

Show that the area of the shaded region OBC is $\frac{1}{6}r^2(3\sqrt{3} - \pi)$. **(6)**

4. *(a)* Sketch on the same diagram the graphs of $y = \sin 2x$ and $y = \tan \frac{x}{2}$ for x in the interval $0 \leq x \leq 360°$. **(4)**

(b) Hence state how many solutions exist to the equation

$$\sin 2x = \tan \frac{x}{2},$$

for x in the interval $0 \leq x \leq 360°$ and give a reason for your answer. **(2)**

5. *(a)* Find the value of a such that

$$\log_a 27 = 3 + \log_a 8.$$ **(3)**

(b) Solve the equation

$$2^{x+3} = 6^{x-1},$$

giving your answer to 3 significant figures. **(4)**

9.

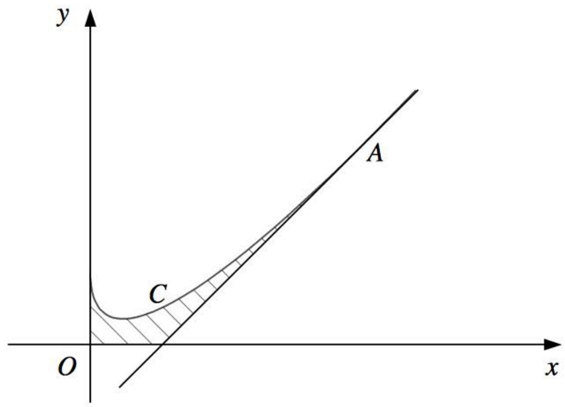

Figure 2

Figure 2 shows the curve C with equation $y = 3x - 4\sqrt{x} + 2$ and the tangent to C at the point A.

Given that A has x-coordinate 4,

(a) show that the tangent to C at A has the equation $y = 2x - 2$. **(6)**

The shaded region is bounded by C, the tangent to C at A and the positive coordinate axes.

(b) Find the area of the shaded region. **(8)**