# C2 Sequences and Series

**1.** June 2010 qu.3

(i) Find and simplify the first four terms in the binomial expansion of $(1 + \frac{1}{2}x)^{10}$ in ascending powers of $x$. [4]

(ii) Hence find the coefficient of $x^3$ in the expansion of $(3 + 4x + 2x^2)(1 + \frac{1}{2}x)^{10}$. [3]

**2.** June 2010 qu.4

A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5n + 1$.

(i) State the values of $u_1$, $u_2$ and $u_3$. [1]

(ii) Evaluate $\sum_{n=1}^{40} u_n$. [3]

Another sequence $w_1, w_2, w_3, \ldots$ is defined by $w_1 = 2$ and $w_{n+1} = 5w_n + 1$.

(iii) Find the value of $p$ such that $u_p = w_3$. [3]

**3.** June 2010 qu.9

A geometric progression has first term $a$ and common ratio $r$, and the terms are all different. The first, second and fourth terms of the geometric progression form the first three terms of an arithmetic progression.

(i) Show that $r^3 - 2r + 1 = 0$. [3]

(ii) Given that the geometric progression converges, find the exact value of $r$. [5]

(iii) Given also that the sum to infinity of this geometric progression is $3 + \sqrt{5}$, find the value of the integer $a$. [4]

**4.** Jan 2010 qu.3

(i) Find and simplify the first four terms in the expansion of $(2 - x)^7$ in ascending powers of $x$. [4]

(ii) Hence find the coefficient of $w^6$ in the expansion of $\left(2 - \frac{1}{4}w^2\right)^7$. [2]

**5.** Jan 2010 qu.8

A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 8$ and $u_{n+1} = u_n + 3$.

(i) Show that $u_5 = 20$. [2]

(ii) The $n$th term of the sequence can be written in the form $u_n = pn + q$. State the values of $p$ and $q$. [2]

(iii) State what type of sequence it is. [1]

(iv) Find the value of $N$ such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$. [5]

**6.** June 2009 qu.2

The tenth term of an arithmetic progression is equal to twice the fourth term. The twentieth term of the progression is 44.

(i) Find the first term and the common difference. [4]

(ii) Find the sum of the first 50 terms. [2]

**7.** Jan 2009 qu.3

A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by $u_n = 24 - \frac{2}{3}n$.

(i) Write down the exact values of $u_1$, $u_2$ and $u_3$. [2]

(ii) Find the value of $k$ such that $u_k = 0$. [2] (iii) Find $\sum_{n=1}^{20} u_n$ . [3]

**8.** Jan 2009 qu.6

A geometric progression has first term 20 and common ratio 0.9.

(i) Find the sum to infinity. [2]

(ii) Find the sum of the first 30 terms. [2]

(iii) Use logarithms to find the smallest value of $p$ such that the $p$th term is less than 0.4. [4]

**9.**

In the binomial expansion of $(k + ax)^4$ the coefficient of $x^2$ is 24.

(i) Given that $a$ and $k$ are both positive, show that $ak = 2$. [3]

(ii) Given also that the coefficient of $x$ in the expansion is 128, find the values of $a$ and $k$. [4]

(iii) Hence find the coefficient of $x^3$ in the expansion. [2]

**10.** Jan 2009 qu.7

Find and simplify the first three terms in the expansion of $(2 - 3x)^6$ in ascending powers of $x$. [4]

**11.** June 2008 qu.2

A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 3$ and $u_{n+1} = 1 - \frac{1}{u_n}$ for $n \geq 1$.

(i) Write down the values of $u_2$, $u_3$ and $u_4$. [3]

(ii) Describe the behaviour of the sequence. [1]

**12.** June 2008 qu.10

Jamie is training for a triathlon, which involves swimming, running and cycling.

- On Day 1, he swims 2 km and then swims the same distance on each subsequent day.
- On Day 1, he runs 2 km and, on each subsequent day, he runs 0.5 km further than on the previous day. (Thus he runs 2.5 km on Day 2, 3 km on Day 3, and so on.)
- On Day 1 he cycles 2 km and, on each subsequent day, he cycles a distance 10% further than on the previous day.

(i) Find how far Jamie runs on Day 15. [2]

(ii) Verify that the distance cycled in a day first exceeds 12 km on Day 20. [3]

(iii) Find the day on which the total distance cycled, up to and including that day, first exceeds 200 km. [4]

(iv) Find the total distance travelled, by swimming, running and cycling, up to and including Day 30. [4]

**13.** Jan 2008 qu.6
A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by $u_n = 2n + 5$, for $n \geq 1$.

(i) Write down the values of $u_1$, $u_2$ and $u_3$. [2]

(ii) State what type of sequence it is. [1]

(iii) Given that $\sum_{n=1}^{N} u_n = 2200$, find the value of $N$. [5]

**14.** Jan 2008 qu.8
The first term of a geometric progression is 10 and the common ratio is 0.8.

(i) Find the fourth term. [2]

(ii) Find the sum of the first 20 terms, giving your answer correct to 3 significant figures. [2]

(iii) The sum of the first $N$ terms is denoted by $S_N$, and the sum to infinity is denoted by $S_\infty$

Show that the inequality $S_\infty - S_N < 0.01$ can be written as $0.8^N < 0.0002$,

and use logarithms to find the smallest possible value of $N$. [7]

**15.** Jan 2008 qu.10
(i) Find the binomial expansion of $(2x + 5)^4$, simplifying the terms. [4]

(ii) Hence show that $(2x + 5)^4 - (2x - 5)^4$ can be written as $320x^3 + kx$,

where the value of the constant $k$ is to be stated. [2]

(iii) Verify that $x = 2$ is a root of the equation $(2x + 5)^4 - (2x - 5)^4 = 3680x - 800$,

and find the other possible values of $x$. [6]

**16.** June 2007 qu.1
A geometric progression $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 15$ and $u_{n+1} = 0.8u_n$ for $n \geq 1$.

(i) Write down the values of $u_2$, $u_3$ and $u_4$. [2] (ii) Find $\sum_{n=1}^{20} u_n$ . [3]

**17.** June 2007 qu.2
Expand $\left(x + \frac{2}{x}\right)^4$ completely, simplifying the terms. [5]

**18.** June 2007 qu.7

(a) In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12915. Find the common difference. [4]

(b) In a geometric progression, the second term is –4 and the sum to infinity is 9. Find the common ratio. [7]

**19.** Jan 2007 qu.1
In an arithmetic progression the first term is 15 and the twentieth term is 72. Find the sum of the first 100 terms. [4]

**20.** Jan 2007 qu.6

(i) Find and simplify the first four terms in the expansion of $(1 + 4x)^7$ in ascending powers of $x$. [4]

(ii) In the expainsion of $(3 + ax)(1 + 4x)^7$ the coefficient of $x^2$ is 1001.

Find the value of $a$. [3]

**21.** June 2006 qu.1
Find the binomial expansion of $(3x – 2)^4$. [4]

**22.** June 2006 qu.2
A sequence of terms $u_1, u_2, u_3$ ,... is defined by $u_1 = 2$ and $u_{n+1} = 1 - u_n$ for $n \geq 1$.

(i) Write down the values of $u_2$, $u_3$ and $u_4$. [2]

(ii) Find $\sum_{n=1}^{100} u_n$ . [3]

**23.** June 2006 qu.6

(i) John aims to pay a certain amount of money each month into a pension fund. He plans to pay £100 in the first month, and then to increase the amount paid by £5 each month, i.e. paying £105 in the second month, £110 in the third month, etc.

If John continues making payments according to this plan for 240 months, calculate

(a) how much he will pay in the final month, [2]

(b) how much he will pay altogether over the whole period. [2]

(ii) Rachel also plans to pay money monthly into a pension fund over a period of 240 months, starting with £100 in the first month. Her monthly payments will form a geometric progression, and she will pay £1500 in the final month.

Calculate how much Rachel will pay altogether over the whole period. [5]

**24.** Jan 2006 qu.1

The 20th term of an arithmetic progression is 10 and the 50th term is 70.

(i) Find the first term and the common difference. [4]

(ii) Show that the sum of the first 29 terms is zero. [2]

(b) how much he will pay altogether over the whole period. [2]

(ii) Rachel also plans to pay money monthly into a pension fund over a period of 240 months, starting with £100 in the first month. Her monthly payments will form a geometric progression, and she will pay £1500 in the final month.

Calculate how much Rachel will pay altogether over the whole period. [5]

**25.** Jan 2006 qu.3

(i) Find the first three terms of the expansion, in ascending powers of $x$, of $(1 - 2x)^{12}$. [3]

(ii) Hence find the coefficient of $x^2$ in the expansion of $(1 + 3x)(1 - 2x)^{12}$. [3]

**26.** Jan 2006 qu.5

In a geometric progression, the first term is 5 and the second term is 4.8.

(i) Show that the sum to infinity is 125. [2]

(ii) The sum of the first $n$ terms is greater than 124. Show that $0.96^n < 0.008$,

and use logarithms to calculate the smallest possible value of $n$. [6]

**27.** June 2005 qu.1

A sequence $S$ has terms $u_1, u_2, u_3$ , ... defined by $u_n = 3n - 1$, for $n \geq 1$.

(i) Write down the values of $u_1$, $u_2$ and $u_3$ and state what type of sequence $S$ is. [3]

(ii) Evaluate $\sum_{n=1}^{100} u_n$ . [3]

**28.** June 2005 qu.6

(i) Find the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^3$, simplifying the terms. [4]

(ii) Hence find $\int \left(x^2 + \frac{1}{x}\right)^3 \, dx$. [4]

**29.** June 2005 qu.8

The amounts of oil pumped from an oil well in each of the years 2001 to 2004 formed a geometric progression with common ratio 0.9. The amount pumped in 2001 was 100 000 barrels.

(i) Calculate the amount pumped in 2004.

It is assumed that the amounts of oil pumped in future years will continue to follow the same geometric progression. Production from the well will stop at the end of the first year in which the amount pumped is less than 5000 barrels.

(ii) Calculate in which year the amount pumped will fall below 5000 barrels. [4]

(iii) Calculate the total amount of oil pumped from the well from the year 2001 up to and including the final year of production. [3]