C2 Integration

1. June 2010 qu.2

(i) Use the trapezium rule, with 3 strips each of width 3, to estimate the area of the region bounded by the curve $y = \sqrt[3]{7+x}$, the x-axis, and the lines $x = 1$ and $x = 10$. Give your answer correct to 3 significant figures. [4]

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate of the area. [1]

2. June 2010 qu.6

(a) Use integration to find the exact area of the region enclosed by the curve $y = x^2 + 4x$, the x-axis and the lines $x = 3$ and $x = 5$. [4]

(b) Find $\int (2 - 6\sqrt{y})\,dy$. [3]

(c) Evaluate $\int_1^{\infty} \frac{8}{x^3}\,dx$. [4]

3. Jan 2010 qu.2

The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$. The curve passes through the distinct points (2, 5) and (p, 5).

(i) Find the equation of the curve. [4]

(ii) Find the value of p. [3]

4. Jan 2010 qu.4

(i) Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for

$\int_3^5 \log_{10}(2+x)\,dx$, giving your answer correct to 3 significant figures. [4]

(ii) Use your answer to part (i) to deduce an approximate value for $\int_3^5 \log_{10}\sqrt{2+x}\,dx$, showing your method clearly. [2]

5. Jan 2010 qu.5

The diagram shows parts of the curves $y = x^2 + 1$ and

$y = 11 - \frac{9}{x^2}$, which intersect at (1, 2) and (3, 10).

Use integration to find the exact area of the shaded region

enclosed between the two curves.

[7]

6. June 2009 qu.4

(i) Find the binomial expansion of $(x^2 - 5)^3$, simplifying the terms. [4]

(ii) Hence find $\int (x^2 - 5)^3 \, dx$. [4]

7. June 2009 qu.6

The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + a$, where a is a constant. The curve passes through the points (–1, 2) and (2, 17). Find the equation of the curve. [8]

8. June 2009 qu.9

(i) Sketch the graph of $y = 4k^x$, where k is a constant such that $k > 1$. State the coordinates of any points of intersection with the axes. [2]

(ii) The point P on the curve $y = 4k^x$ has its y-coordinate equal to $20k^2$. Show that the x-coordinate of P may be written as $2 + \log_k 5$. [4]

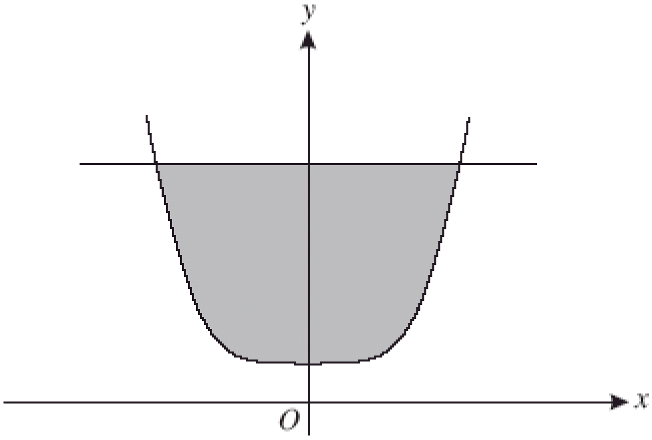
(iii) (a) Use the trapezium rule, with two strips each of width $\frac{1}{2}$, to find an expression for the approximate value of $\int_0^1 4k^x \, dx$. [3]

(b) Given that this approximate value is equal to 16, find the value of k. [3]

9. Jan 2009 qu.1

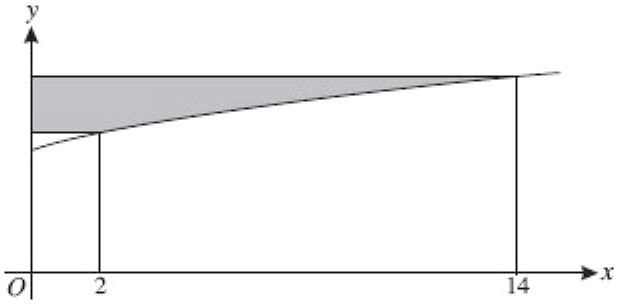
Find (i) $\int (x^3 + 8x - 5) dx$, [3] (ii) $\int 12\sqrt{x} \, dx$. [3]

10. Jan 2009 qu.4



The diagram shows the curve $y = x^4 + 3$ and the line $y = 19$ which intersect at (–2, 19) and (2, 19). Use integration to find the exact area of the shaded region enclosed by the curve and the line. [7]

11. June 2008 qu.5



The diagram shows the curve $y = 3 + \sqrt{x+2}$.

The shaded region is bounded by the curve, the y-axis, and two lines parallel to the x-axis which meet the curve where $x = 2$ and $x = 14$.

(i) Show that the area of the shaded region is given by $\int_5^7 (y^2 - 6y + 7)\,dy$. [3]

(ii) Hence find the exact area of the shaded region. [4]

12. June 2008 qu.7

(a) Find $\int x^3(x^2 - x + 5)\,dx$. [4]

(b) (i) Find $\int 18x^{-4}\,dx$. [2]

(ii) Hence evaluate $\int_2^{\infty} 18x^{-4}\,dx$. [2]

13. June 2008 qu.9

(b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for $\int_0^1 \cos x\,dx$, where x is in radians. Give your answer correct to 3 significant figures. [4]

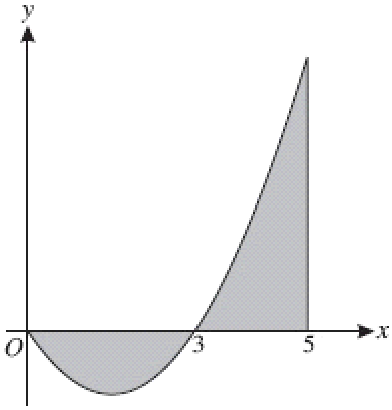
14. Jan 2008 qu.2

Use the trapezium rule, with 3 strips each of width 2, to estimate the value of $\int_1^7 \sqrt{x^2+3}\,dx$. 4]

15. Jan 2008 qu.5

The gradient of a curve is given by $\frac{dy}{dx} = 12\sqrt{x}$. The curve passes through the point (4, 50). Find the equation of the curve. [6]

16. Jan 2008 qu.7



The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

(i) Explain why $\int_0^5 \left(x^2 - 3x\right) dx$ does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

17. June 2007 qu.4

The diagram shows the curve $y = \sqrt{4x + 1}$.

(i) Use the trapezium rule, with strips of width 0.5, to find an approximate value for the area of the region bounded by the curve $y = \sqrt{4x+1}$, the x-axis, and the lines $x = 1$ and $x = 3$.Give your answer correct to 3 significant figures. [4]

(ii) State with a reason whether this approximation is an under-estimate or an over-estimate. [2]

18. June 2007 qu.6

(a) (i) Find $\int x(x^2 - 4)\, dx$. [3]

(ii) Hence evaluate $\int_1^6 x(x^2 - 4) dx$. [2]

(b) Find $\int \frac{6}{x^3} dx$. [3]

19. Jan 2007 qu.3

(i) Find $\int (4x - 5)\,dx$ [2]

(ii) The gradient of a curve is given by $\frac{dy}{dx} = 4x - 5$. The curve passes through the point (3, 7). Find the equation of the curve. [3]

20. Jan 2007 qu.5

(b) Use the trapezium rule, with two strips of width 3, to find an approximate value for

$\int_3^9 \log_{10} x\,dx$ giving your answer correct to 3 significant figures. [4]

21. Jan 2007 qu.10

The diagram shows the graph of $y = 1 - 3x^{-\frac{1}{2}}$.

(i) Verify that the curve intersects the x-axis at (9, 0). [1]

(ii) The shaded region is enclosed by the curve, the x-axis and the line $x = a$ (where $a > 9$).Given that the area of the shaded region is 4 square units, find the value of a. [9]

22. June 2006 qu.3

The gradient of a curve is given by $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$, and the curve passes through the point (4, 5). Find the equation of the curve. [6]

23. June 2006 qu.4

The diagram shows the curve $y = 4 - x^2$ and the line $y = x + 2$.

(i) Find the x-coordinates of the points of intersection of the curve and the line. [2]

(ii) Use integration to find the area of the shaded region bounded by the line and the curve. [6]

24. June 2006 qu.9

(i) Sketch the curve $y = \left(\frac{1}{2}\right)^x$, and state the coordinates of any point where the curve crosses an axis. [3]

(ii) Use the trapezium rule, with 4 strips of width 0.5, to estimate the area of the region bounded by the curve $y = \left(\frac{1}{2}\right)^x$, the axes, and the line $x = 2$. [4]

(iii) The point P on the curve $y = \left(\frac{1}{2}\right)^x$ has y-coordinate equal to $\frac{1}{6}$.

Prove that the x-coordinate of P may be written as $1 + \frac{\log_{10} 3}{\log_{10} 2}$. [4]

25. Jan 2006 qu.6

(a) Find $\int (x^{\frac{1}{2}} + 4)\,dx$. [4]

(b) (i) Find the value, in terms of a, of $\int_1^a 4x^{-2}\,dx$, where a is a constant greater than 1. [3]

(ii) Deduce the value of $\int_1^\infty 4x^{-2}\,dx$. [1]

26. Jan 2006 qu.8

The cubic polynomial $2x^3 + kx^2 - x + 6$ is denoted by $f(x)$. It is given that $(x + 1)$ is a factor of $f(x)$.

(i) Show that $k = -5$, and factorise $f(x)$ completely. [6]

(ii) Find $\int_{-1}^{2} f(x)\, dx$. [4]

(iii) Explain with the aid of a sketch why the answer to part (ii) does not give the area of the region between the curve $y = f(x)$ and the x-axis for $-1 \le x \le 2$. [2]

27. June 2005 qu.3

(i) Find $\int (2x+1)(x+3)\, dx$ [4]

(ii) Evaluate $\int_{0}^{9} \frac{1}{\sqrt{x}}\, dx$ [3]