1. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x})\,dx.$$

(Total 5 marks)

2. Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(Total 4 marks)

3. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,dx$.

(4)
(Total 7 marks)

4. Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right)dx$

(Total 5 marks)

5. Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$,

(2)

(b) evaluate $\int_1^3 y \, \mathrm{d}x$.

(4)
(Total 6 marks)

6. Given that $y = 6x - \dfrac{4}{x^2}$, $x \neq 0$,

(a) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$,

(2)

(b) find $\int y \; \mathrm{d}x$.

(3)
(Total 5 marks)

7. (a) Expand $(2\sqrt{x} + 3)^2$.

(2)

(b) Hence evaluate $\int_1^2 (2\sqrt{x} + 3)^2 \; \mathrm{d}x$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(5)
(Total 7 marks)

8. (a) Find $\int \left(3 + 4x^3 - \frac{2}{x^2}\right) dx$.

(3)

(b) Hence evaluate

$$\int_1^2 \left(3 + 4x^3 - \frac{2}{x^2}\right) dx.$$

(2)
(Total 5 marks)

1. $\int \left(2x + 3x^{\frac{1}{2}}\right) dx = \frac{2x^2}{2} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ M1 A1A1

$\int_1^4 \left(2x + 3x^{\frac{1}{2}}\right) dx = \left[x^2 + 2x^{\frac{3}{2}}\right]_1^4 = (16 + 2 \times 8) - (1 + 2)$ M1

$= 29$ (29 + C scores A0) A1 5

<u>Note</u>

1st M1 for attempt to integrate $x \to kx^2$ or $x^{\frac{1}{2}} \to kx^{\frac{3}{2}}$.

1st A1 for $\frac{2x^2}{2}$ or a simplified version.

2nd A1 for $\frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ or $\frac{3x\sqrt{x}}{\left(\frac{3}{2}\right)}$ or a simplified version.

Ignore + C, if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A0.

2nd M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation).

Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear.

<u>No working:</u>
The answer 29 with no working scores M0A0A0M1A0 (1 mark).

[5]

2. $\int x^{-\frac{1}{2}} \mathrm{d}x = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (Or equivalent, such as $2x^{\frac{1}{2}}$, or $2\sqrt{x}$) M1A1

$\left[\dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_1^8 = 2\sqrt{8} - 2 = -2 + 4\sqrt{2}$ [or $4\sqrt{2} - 2$, or $2(2\sqrt{2} - 1)$, or $2(-1 + 2\sqrt{2})$] M1A1 4

1st M1: $x^{-\frac{1}{2}} \to kx^{\frac{1}{2}}$, $k \neq 0$.

2nd M: Substituting limits 8 and 1 into a 'changed' function (i.e. not $\dfrac{1}{\sqrt{x}}$ or $x^{-\frac{1}{2}}$), and subtracting, either way round.

2nd A: This final mark is still scored if $-2 + 4\sqrt{2}$ is reached via a decimal.

N.B. Integration constant $+C$ may appear, e.g.

$$\left[\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + C\right]_1^8 = (2\sqrt{8} + C) - (2 + C) = -2 + 4\sqrt{2} \text{ (Still full marks)}$$

<u>But</u>... a final answer such as $-2 + 4\sqrt{2} + C$ is A0.

N.B. It will sometimes be necessary to 'ignore subsequent working' (isw) after a correct form is seen, e.g. $\int x^{-\frac{1}{2}} \mathrm{d}x = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (M1 A1),

followed by incorrect simplification $\int x^{-\frac{1}{2}} \mathrm{d}x = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = \dfrac{1}{2}x^{\frac{1}{2}}$

(still M1 A1).... The second M mark is still available for substituting 8 and 1 into $\dfrac{1}{2}x^{\frac{1}{2}}$ and subtracting.

[4]

3. (a) $f'(x) = 3x^2 + 6x$ B1

$f''(x) = 6x + 6$ M1, A1cao 3

Acceptable alternatives include
$3x^2 + 6x^1$; $3x^2 + 3\times 2x$; $3x^2 + 6x + 0$

Ignore LHS (e.g. use [whether correct or not] of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$)

$3x^2 + 6x + c$ or $3x^2 + 6x$ + constant (i.e. the written word constant) is B0 B1

M1 Attempt to differentiate their $f'(x)$; $x^n \to x^{n-1}$.
$x^n \to x^{n-1}$ seen in at least one of the terms. Coefficient of $x^{\cdots}$ ignored for the method mark.
$x^2 \to x^1$ and $x \to x^0$ are acceptable. M1

Acceptable alternatives include
$6x^1 + 6x^0$; $3 \times 2x + 3 \times 2$
$6x + 6 + c$ or $6x + 6$ + constant is A0 A1 cao

<u>Examples</u>

$f''(x) = 3x^2 + 6x$	B1 M0 A0		$f'(x) = x^2 + 3x$ $f''(x) = x + 3$	B0 M1 A0	
$f'(x) = 3x^2 + 6x$ $f''(x) = 6x$	B1 M1 A0		$x^3 + 3x^2 + 5$ $= 3x^2 + 6x$ $= 6x + 6$	 B1 M1 A1	
$y = x^3 + 3x^2 + 5$			$f'(x) = 3x^2 + 6x + 5$	B0	
$\frac{dy}{dx} = 3x^2 + 3x$	B0		$f''(x) = 6x + 6$	M1 A1	
$\frac{d^2y}{dx^2} = 6x + 3$	M1A0		$f'(x) = 3x^2 + 6x$ $f''(x) = 6x + 6 + c$	B1 M1 A0	
$f'(x) = 3x^2 + 6x + c$ $f''(x) = 6x + 6$	B0 M1 A1				

(b) $\int (x^3 + 3x^2 + 5)dx = \frac{x^4}{4} + \frac{3x^3}{3} + 5x$ M1, A1

$\left[\frac{x^4}{4} + x^3 + 5x\right]_1^2 = 4 + 8 + 10 - (\frac{1}{4} + 1 + 5)$ M1

$= 15\frac{3}{4}$ o.e. A1 4

Attempt to integrate f(x); $x^n \to x^{n+1}$ Ignore incorrect notation (e.g. inclusion of integral sign)	M1
o.e. Acceptable alternatives include $\frac{x^4}{4} + x^3 + 5x; \frac{x^4}{4} + \frac{3x^3}{3} + 5x^1; \frac{x^4}{4} + \frac{3x^3}{3} + 5x + c; \int \frac{x^4}{4} + \frac{3x^3}{3} + 5x$ N.B. If the candidate has written the integral (either $\frac{x^4}{4} + \frac{3x^3}{3} + 5x$ or what they think is the integral) in part (a), it may not be rewritten in (b), but the marks may be awarded if the integral is used in (b).	A1
Substituting 2 and 1 into any function other than $x^3 + 3x^2 + 5$ and subtracting either way round. So using their f′(x) or f″(x) or ∫ their f′(x)dx or ∫their f″(x) dx will gain the M mark (because none of these will give $x^3 + 3x^2 + 5$). Must substitute for all x s but could make a slip. $4 + 8 + 10 - \frac{1}{4} + 1 + 5$ (for example) is acceptable for evidence of subtraction ('invisible' brackets).	M1
o.e. (e.g. $15\frac{3}{4}$, 15.75, $\frac{63}{4}$) Must be a single number (so $22 - 6\frac{1}{4}$ is A0).	A1

Answer only is M0A0M0A0

<u>Examples</u>

$\frac{x^4}{4} + x^3 + 5x + c$	M1A1	$\frac{x^4}{4} + x^3 + 5x + c$	M1 A1
$4 + 8 + 10 + c - (\frac{1}{4} + 1 + 5 + c)$	M1	$x = 2, 22 + c$	
$= 15\frac{3}{4}$	A1	$x = 1, 6\frac{1}{4} + c$ (no subtraction)	M0 A0

$\int_1^2 f(x)dx = 2^3 + 3 \times 2^2 + 5 - (1 + 3 + 5)$ M0A0, M0

$= 25 - 9$

$= 16$ A0

(Substituting 2 and 1 into $x^3 + 3x^2 + 5$, so 2nd M0)

$\int_1^2 (6x+6)\mathrm{d}x = \left[3x^2+6x\right]_1^2$ M0 A0 $\int_1^2 (3x^2+6x)\mathrm{d}x = \left[x^3+3x^2\right]_1^2$ M0 A0

$= 12 + 12 - (3 + 6)$ M1 A0 $= 8 + 12 - (1 + 3)$ M1 A0

$\frac{x^4}{4} + x^3 + 5x$ M1 A1

$\frac{2^4}{4} + 2^3 + 5 \times 2 - \frac{1^4}{4} + 1^3 + 5$ M1

(one negative sign is sufficient for evidence of subtraction)

$= 22 - 6\frac{1}{4} = 15\frac{3}{4}$ A1

(allow 'recovery', implying student was using 'invisible brackets')

(a) $f(x) = x^3 + 3x^2 + 5$

$f''(x) = \frac{x^4}{4} + x^3 + 5x$ B1M0A0

(b) $\frac{2^4}{4} + 2^3 + 5 \times 2 - \frac{1^4}{4} - 1^3 - 5$ M1A1M1

$= 15\frac{3}{4}$ A1

The candidate has written the integral in part (a). It is not rewritten in (b), but the marks may be awarded as the integral is used in (b).

[7]

4. $\int (3x^2 + 5 + 4x^{-2})\mathrm{d}x = \frac{3x^3}{3} + 5x + \frac{4x^{-1}}{-1}$ $(= x^3 + 5x - 4x^{-1})$ M1 A1 A1

$\left[x^3\ 5x - 4x^{-1}\right]_1^2 = (8 + 10 - 2) - (1 + 5 - 4), = 14$ M1, A1 5

Integration:

Accept any correct version, simplified or not.

All 3 terms correct: M1 A1 A1,
Two terms correct: M1 A1 A0,
One power correct: M1 A0 A0.

The given function must be integrated to score M1, and not e.g. $3x^4 + 5x^2 + 4$.

Limits:

M1: Substituting 2 and 1 into a 'changed function' and subtracting, either way round.

[5]

5. (a) $\frac{dy}{dx} = 4x + 18x^{-4}$ — $x^n \mapsto x^{n-1}$ — M1 A1 — 2

(b) $\int \left(2x^2 - 6x^{-3}\right) dx = \frac{2}{3}x^3 + 3x^{-2}$ — $x^n \mapsto x^{n+1}$ — M1 A1

$[...]_1^3 = \frac{2}{3} \times 3^3 + \frac{3}{9} - \left(\frac{2}{3} + 3\right)$ — M1

$= 14\frac{2}{3}$ $\quad \frac{44}{3}, \frac{132}{9}$ or equivalent — A1 — 4

[6]

6. (a) $\frac{dy}{dx} = 6 + 8x^{-3}$ — M1 A1 — 2

M1 is for $x^n \to x^{n-1}$ in at least one term, 6 or x^{-3} is sufficient.
A1 is fully correct answer.
Ignore subsequent working.

(b) $\int y\,dx = \frac{6x^2}{2} + 4x^{-1} + C$ — M1 A1 A1 — 3

M1: Correct power of x in at least one term (C sufficient)

First A1: $\frac{6x^2}{2} + C$

Second A1: $+ 4x^{-1}$

[5]

7. (a) $4x + 9, +12\sqrt{x}$ (Allow $6\sqrt{x} + 6\sqrt{x}$) — B1, B1 — 2
Allow $(2\sqrt{x})^2$ or $4\sqrt{x^2}$ only if later work justifies understanding.

(b) $\int (4x + 12x^{1/2} + 9)dx = 2x^2 + 8x^{3/2} + 9x$ (ft dep. on 3 terms) — M1 A1 ft

$[.....]_1^2 = (8 + (8 \times 2^{3/2}) + 18) - (2 + 8 + 9)$ — M1

$2^{3/2} = 2\sqrt{2}$ (seen or implied) — B1

$= 7 + 16\sqrt{2}$ — A1 — 5

Special case: Misread as $2\sqrt{(x + 3)}$

(a) B1 for $4x + 12$
(b) Just the two M marks are available.

[7]

8. (a) $\int (3 + 4x^3 - \frac{2}{x^2})\,dx = 3x + x^4 + \frac{2}{x} + c$ M1 A2 (1, 0) 3

(b) $\int_1^2 (3 + 4x^3 - \frac{2}{x^2})\,dx = \left[3x + x^4 + \frac{2}{x}\right]_1^2$

$= (6 + 16 + 1) - (3 + 1 + 2)$ M1

$= 17$ A1 2

[5]

1. While most candidates gained method marks for their integration attempt, some did not realise that $\sqrt{x}$ was $x^{\frac{1}{2}}$, or had difficulty in simplifying $3x^{\frac{3}{2}} \div \left(\frac{3}{2}\right)$. More common was the inability to evaluate the definite integral correctly, with $2 \times 4^{\frac{3}{2}}$ causing particular problems. Overall, however, it was common to see full marks scored.

2. Although many candidates scored full marks on this question, others had difficulty dealing with $\frac{1}{\sqrt{x}}$. This was sometimes misinterpreted as $x^{\frac{1}{2}}$ or x^{-1}, leading to incorrect integration but still allowing the possibility of scoring a method mark for use of limits. Sometimes there was no integration attempt at all and the limits were simply substituted into $\frac{1}{\sqrt{x}}$. The candidates who performed the integration correctly were usually able to deal with their surds and proceeded score full marks, although a few left their answer as $-1 + 2\sqrt{8}$. Occasionally the answer was given as a decimal.

3. Part (a) was answered well with many correct solutions. A few candidates integrated f(x). Some candidates had difficulty differentiating the constant term. The most common incorrect solution was $f''(x) = 6x$. In part (b) a few candidates used f(x) or f′(x) as their integral. However, most integrated successfully and substituted accurately. Occasional arithmetic slips were seen.

4. This standard test of definite integration was handled well by the vast majority of candidates. Mistakes, where made, tended to be in the integration of x^{-2}, although errors in simple arithmetic sometimes spoilt otherwise correct solutions. Candidates who differentiated were only able to pick up one method mark, for the substitution of limits.

5. The general standard of calculus displayed throughout the paper was excellent and full marks were common on this question. A few candidates took the negative index in the wrong direction, differentiating x^{-3} to obtain $-2x^{-2}$ and integrating x^{-3} to obtain $-\frac{x^{-4}}{4}$.

6. **Pure Mathematics P1**

The vast majority of candidates gained the method marks and many went on to score four or five marks; the four mark score was usually due to the omission of the arbitary constant in part (b).

Differentiation and integration of $-\frac{4}{x^2}$ caused problems for some candidates, and those who wrote y as $\frac{6x^3 - 4}{x^2}$ before differentiating were usually defeated.

Providing $8x^{-3}$ in part (a) and $+ 4x^{-1}$ in part (b) had been seen, subsequent errors such as writing these as $\frac{1}{8x^3}$ or $\frac{1}{4x}$ respectively, were not penalised, but it should be noted that such errors with indices were quite common.

Core Mathematics C1

This was another straightforward question for many candidates and the principles of differentiation and integration were understood by most. The negative power caused problems for some again, they realized that the term $-\frac{4}{x^2}$ needed to be written as $-4x^{-2}$ but did not always apply the rules successfully. In part (b) some could not simplify $-\frac{4x^{-1}}{-1}$ to $+4x^{-1}$ and a sizeable minority lost a mark for failing to include a constant of integration.

7. Most candidates made good attempts to expand the expression in part (a), although a few did not recognise that $\sqrt{x^2}$ is equivalent to x and some gave $(2\sqrt{x})^2 = 2x$. Integration techniques in part (b) were well known. The most common cause of mark loss in this part was the inability to express the final answer in the required surd form, with $2^{3/2}$ not being recognised as 2Ö2, and $8 \times 2^{3/2}$ sometimes "simplified" to $16^{3/2}$.

8. No Report available for this question.