1. $$f(x) = 3x^3 - 5x^2 - 58x + 40$$

(a) Find the remainder when f (x) is divided by (x – 3)

(2)

Given that (x – 5) is a factor of f (x),

(b) find all the solutions of f (x) = 0.

(5)
(Total 7 marks)

2. $$f(x) = 2x^3 + ax^2 + bx - 6$$

where a and b are constants.
When f(x) is divided by (2x – 1) the remainder is –5.
When f(x) is divided by (x + 2) there is no remainder.

(a) Find the value of a and the value of b.

(6)

(b) Factorise f(x) completely.

(3)
(Total 9 marks)

3. $$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(a) Write down the value of f (k).

(1)

When f (x) is divided by (x – 2) the remainder is 4

(b) Find the value of k.

(2)

(c) Factorise f (x) completely.

(3)
(Total 6 marks)

4. $$\mathrm{f}(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when f(x) is divided by (x – 2) is equal to the remainder when f(x) is divided by (x + 1).

(a) Find the value of a.

(5)

Given that (x + 3) is a factor of f(x),

(b) find the value of b.

(3)
(Total 8 marks)

5. $$\mathrm{f}(x) = 2x^3 - 3x^2 - 39x + 20$$

(a) Use the factor theorem to show that (x + 4) is a factor of f (x).

(2)

(b) Factorise f (x) completely.

(4)
(Total 6 marks)

6. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x_3 - 2x_2 - 4x + 8 = 0.$$

(4)
(Total 7 marks)

7. $$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

(a) Find the remainder when $f(x)$ is divided by $(x - 2)$.

(2)

Given that $(x + 2)$ is a factor of $f(x)$,

(b) factorise $f(x)$ completely.

(4)
(Total 6 marks)

8. $$f(x) = x^3 + 4x^2 + x – 6.$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0.$$

(1)
(Total 7 marks)

9. $$f(x) = 2x^3 + 3x^2 - 29x - 60.$$

(a) Find the remainder when $f(x)$ is divided by $(x + 2)$.

(2)

(b) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(c) Factorise $f(x)$ completely.

(4)
(Total 8 marks)

10. $f(x) = 2x^3 - x^2 + ax + b$, where a and b are constants.

It is given that $(x - 2)$ is a factor of $f(x)$.

When $f(x)$ is divided by $(x + 1)$, the remainder is 6.

Find the value of a and the value of b.

(Total 7 marks)

11. $f(x) = 2x^3 + x^2 - 5x + c$, where c is a constant.

Given that $f(1) = 0$,

(a) find the value of c,

(2)

(b) factorise $f(x)$ completely,

(4)

(c) find the remainder when $f(x)$ is divided by $(2x - 3)$.

(2)
(Total 8 marks)

12. A function f is defined as

$$f(x) = 2x^3 - 8x^2 + 5x + 6, \quad x \in \mathbb{R}.$$

Using the remainder theorem, or otherwise, find the remainder when $f(x)$ is divided by

(a) $(x - 2)$,

(2)

(b) $(2x + 1)$.

(2)

(c) Write down a solution of $f(x) = 0$.

(1)
(Total 5 marks)

13. (a) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + x^2 - 25x + 12$.

(2)

(b) Factorise $2x^3 + x^2 - 25x + 12$ completely.

(4)
(Total 6 marks)

14. $$f(x) = 2x^3 - x^2 + 2x - 16.$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

Given that $f(x) = (x - 2)(2x^2 + bx + c)$,

(b) find the values of b and c.

(3)

(c) Hence prove that $f(x) = 0$ has only one real solution.

(3)
(Total 8 marks)

15. $$f(x) = 2ax^3 - ax^2 - 3x + 7,$$

where a is a constant.

Given that the remainder when $f(x)$ is divided by $(x + 2)$ is -3,

(a) find the value of a,

(3)

(b) find the remainder when $f(x)$ is divided by $(2x - 1)$.

(2)
(Total 5 marks)

16. $f(x) = x^3 - 2x^2 + ax + b$, where a and b are constants.

When $f(x)$ is divided by $(x - 2)$, the remainder is 1.

When $f(x)$ is divided by $(x + 1)$, the remainder is 28.

(a) Find the value of a and the value of b.

(6)

(b) Show that $(x - 3)$ is a factor of $f(x)$.

(2)
(Total 8 marks)

17.

$f(x) = x^3 + (p + 1)x^2 - 18x + q$, where p and q are integers.

Given that $(x - 4)$ is a factor of $f(x)$,

(a) show that $16p + q + 8 = 0$.

(3)

Given that $(x + p)$ is also a factor of $f(x)$, and that $p > 0$,

(b) show that $p^2 + 18p + q = 0$.

(3)

(c) Hence find the value of p and the corresponding value of q.

(5)

(d) Factorise $f(x)$ completely.

(2)
(Total 13 marks)

18.

$$f(x) = (x^2 + p)(2x + 3) + 3,$$

where p is a constant.

(a) Write down the remainder when $f(x)$ is divided by $(2x + 3)$.

(1)

Given that the remainder when $f(x)$ is divided by $(x – 2)$ is 24,

(b) prove that $p = –1$,

(2)

(c) factorise $f(x)$ completely.

(4)
(Total 7 marks)

19.

$$f(x) = x^3 – 19x – 30.$$

(a) Show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)
(Total 6 marks)

20. $f(x) = 6x^3 + px^2 + qx + 8$, where p and q are constants.

Given that $f(x)$ is exactly divisible by $(2x – 1)$, and also that when $f(x)$ is divided by $(x – 1)$ the remainder is $–7$,

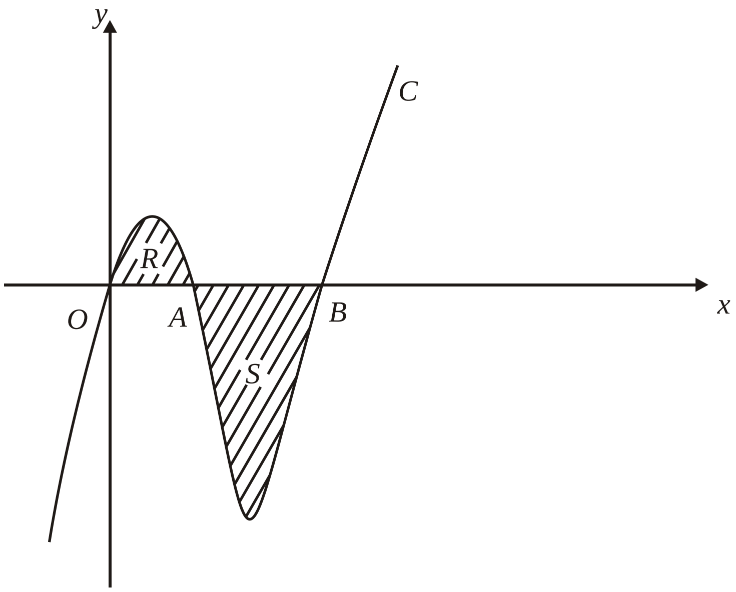
(a) find the value of p and the value of q.

(6)

(b) Hence factorise f(*x*) completely.

(3)
(Total 9 marks)

21.



The diagram above shows part of the curve *C* with equation $y = f(x)$, where

$$f(x) = x^3 - 6x^2 + 5x.$$

The curve crosses the *x*-axis at the origin *O* and at the points *A* and *B*.

(a) Factorise f(*x*) completely.

(3)

(b) Write down the *x*-coordinates of the points *A* and *B*.

(1)

(c) Find the gradient of *C* at *A*.

(3)

The region R is bounded by C and the line OA, and the region S is bounded by C and the line AB.

(d) Use integration to find the area of the combined regions R and S, shown shaded in the diagram above.

(7)
(Total 14 marks)

22. $$f(x) = px^3 + 6x^2 + 12x + q.$$

Given that the remainder when f(x) is divided by (x – 1) is equal to the remainder when f(x) is divided by (2x + 1),

(a) find the value of p.

(4)

Given also that q = 3, and p has the value found in part (a),

(b) find the value of the remainder.

(1)
(Total 5 marks)

23. $$f(n) = n^3 + pn^2 + 11n + 9,\text{ where } p \text{ is a constant.}$$

(a) Given that f(n) has a remainder of 3 when it is divided by (n + 2), prove that $p = 6$.

(2)

(b) Show that f(n) can be written in the form $(n + 2)(n + q)(n + r) + 3$, where q and r are integers to be found.

(3)

(c) Hence show that f(n) is divisible by 3 for all positive integer values of n.

(2)
(Total 7 marks)

24.

$$f(x) = ax^3 + 3x^2 + bx + 1,$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$ there is a remainder of 5. When $f(x)$ is divided by $(x + 2)$ there is a remainder of -1.

Find the value of a and the value of b.

(Total 5 marks)

25. $f(x) = 2x^3 - x^2 + px + 6$,

where p is a constant.

Given that $(x - 1)$ is a factor of $f(x)$, find

(a) the value of p,

(2)

(b) the remainder when $f(x)$ is divided by $(2x + 1)$.

(2)

(Total 4 marks)

1. (a) Attempting to find $\text{f}(3)$ or $\text{f}(-3)$ M1

$\text{f}(3) = 3(3)^3 - 5(3)^2 - (58 \times 3) + 40 = 81 - 45 - 174 + 40 = -98$ A1 2

Note

Alternative (long division):

Divide by $(x - 3)$ to get $(3x^2 + ax + b)$, $a \neq 0$, $b \neq 0$. M1

$(3x^2 + 4x - 46)$, and –98 seen. A1

(If continues to say 'remainder = 98', isw)

'Grid' method

3	3	–5	–58	40
	0	9	12	–138
	3	4	–46	–98

(b) $\{3x^3 - 5x^2 - 58x + 40 = (x-5)\}\ (3x^2 + 10x - 8)$ M1 A1

Attempt to factorise 3-term quadratic, or to use the quadratic formula (see general principles at beginning of scheme). This mark may be implied by the correct solutions to the quadratic. M1

$(3x-2)(x+4) = 0$ $x = ...$ or $x = \frac{-10 \pm \sqrt{100+96}}{6}$ A1 ft

$\frac{2}{3}$ (or exact equiv.), -4, 5 (Allow 'implicit' solns, e.g. f(5) = 0, etc.) A1 5

Completely correct solutions without working: full marks.

Note

1st M requires use of $(x - 5)$ to obtain $(3x^2 + ax + b)$, $a \neq 0$, $b \neq 0$. (Working need not be seen… this could be done 'by inspection'.)

'Grid' method

3	3	–5	–58	40	
	0	15	50	–40	
	3	10	–8	0	$\rightarrow (3x^2 + 10x - 8)$

2nd M for the attempt to factorise their 3-term quadratic, or to solve it using the quadratic formula.

Factorisation: $(3x^2 + ax + b) = (3x + c)(x + d)$, where $|cd| = |b|$.

A1ft: Correct factors for their 3-term quadratic followed by a solution (at least one value, which might be incorrect), or numerically correct expression from the quadratic formula for their 3-term quadratic.

Note therefore that if the quadratic is correctly factorised but no solutions are given, the last 2 marks will be lost.

Alternative (first 2 marks):

$(x - 5)(3x^2 + ax + b) = 3x^3 + (a - 15)x^2 + (b - 5a)x - 5b = 0$,

then compare coefficients to find values of a and b. M1

$a = 10$, $b = -8$ A1

Alternative 1: (factor theorem)

M1: Finding that $f(-4) = 0$

A1: Stating that $(x + 4)$ is a factor.

M1: Finding third factor $(x - 5)(x + 4)(3x \pm 2)$.

A1: Fully correct factors (no ft available here) followed by a solution, (which might be incorrect).

A1: All solutions correct.

Alternative 2: (direct factorisation)

M1: Factors $(x - 5)(3x + p)(x + q)$ A1: $pq = -8$

M1: $(x - 5)(3x \pm 2)(x \pm 4)$

Final A marks as in Alternative 1.

Throughout this scheme, allow $\left(x \pm \frac{2}{3}\right)$ as an alternative to $(3x \pm 2)$.

[7]

2. (a) $f\left(\tfrac{1}{2}\right) = 2 \times \frac{1}{8} + a \times \frac{1}{4} + b \times \frac{1}{2} - 6$ M1

$f\left(\tfrac{1}{2}\right) = -5 \Rightarrow \tfrac{1}{4}a + \tfrac{1}{2}b = \tfrac{3}{4}$ or $a + 2b = 3$ A1

$f(-2) = -16 + 4a - 2b - 6$ M1

$f(-2) = 0 \Rightarrow 4a - 2b = 22$ A1

Eliminating one variable from 2 linear simultaneous equations in a and b M1

$a = 5$ and $b = -1$ A1 6

Note

1st M1 for attempting f($\pm\frac{1}{2}$) Treat the omission of the –5 here as a slip and allow the M mark.

1st A1 for first correct equation in *a* and *b* simplified to three non zero terms (needs –5 used)

s.c. If it is not simplified to three terms but is correct and is then used correctly with second equation to give correct answers- this mark can be awarded later.

2nd M1 for attempting f(∓ 2)

2nd A1 for the second correct equation in *a* and *b*. simplified to three terms (needs 0 used) s.c. If it is not simplified to three terms but is correct and is then used correctly with first equation to give correct answers – this mark can be awarded later.

3rd M1 for an attempt to eliminate one variable from 2 linear simultaneous equations in *a* and *b*

3rd A1 for both $a = 5$ and $b = -1$ (Correct answers here imply previous two A marks)

Alternative;

M1 for dividing by $(2x - 1)$, to get $x^2 + \left(\frac{a+1}{2}\right)x +$ constant **with remainder as a function of *a* and *b***, and A1 as before for equations stated in scheme .

M1 for dividing by $(x + 2)$, to get $2x^2 + (a - 4)x$... (No need to see remainder as it is zero and comparison of coefficients may be used) with A1 as before

(b)	$2x^3 + 5x^2 - x - 6 = (x+2)(2x^2 + x - 3)$	M1	
	$= (x + 2)(2x + 3)(x - 1)$	M1A1	3

NB $(x+2)(x+\frac{3}{2})(2x-2)$ is A0 But $2(x+2)(x+\frac{3}{2})(x-1)$ is A1

Note

1st M1 for attempt to divide by $(x+2)$ leading to a 3TQ beginning with correct term usually $2x^2$

2nd M1 for attempt to factorize their quadratic provided no remainder

A1 is cao and needs all three factors

Ignore following work (such as a solution to a quadratic equation).

Alternative;

M1 for finding second factor correctly by factor theorem, usually $(x - 1)$
M1 for using two known factors to find third factor, usually $(2x \pm 3)$
Then A1 for correct factorisation written as product $(x + 2)(2x + 3)(x - 1)$

[9]

3.	(a)	$f(k) = -8$	B1	1
	(b)	$f(2) = 4 \Rightarrow 4 = (6 - 2)(2 - k) - 8$	M1	
		So $k = -1$	A1	2

Note

M1 for substituting $x = 2$ (not $x = -2$) and equating to 4 to form an equation in k. If the expression is expanded in this part, condone 'slips'for this M mark Treat the omission of the – 8 here as a 'slip' and allow the M mark.

Beware:
Substituting $x = -2$ and equating to 0 (M0 A0) also gives $k = -1$.

Alternative;
M1 for dividing by $(x - 2)$, to get $3x$ + (function of k), with remainder as a function of k, and equating the remainder to 4. [Should be $3x + (4 - 3k)$, remainder $4k$].

No working:
$k = -1$ with no working scores M0 A0.

(c) $f(x) = 3x^2 - (2 + 3k)\,x + (2k - 8) = 3x^2 + x - 10$ M1

$= (3x - 5)(x + 2)$ M1A1 3

<u>Note</u>

1st M1 for multiplying out <u>and</u> substituting their (constant) value of k (in either order). The multiplying-out may occur earlier. Condone, for example, sign slips, but if the 4 (from part (b)) is included in the f(x) expression, this is M0. The 2nd M1 is still available.

2nd M1 for an attempt to factorise their three term quadratic (3TQ).

A1 The correct answer, as a <u>product of factors</u>, is required.

Allow $3\left(x - \frac{5}{3}\right)(x + 2)$

Ignore following work (such as a solution to a quadratic equation). If the 'equation' is solved but factors are never seen, the 2nd M is not scored.

[6]

4. (a) $f(2) = 16 + 40 + 2a + b$ **or** $f(-1) = 1 - 5 - a + b$ M1 A1

Finds 2nd remainder and equates to 1st $\Rightarrow$ $16 + 40 + 2a + b = 1 - 5 - a + b$ M1 A1

$a = -20$ A1cso 5

<u>Notes:</u>

M1: Attempts f(± 2) or f(± 1)
A1 is for the answer shown (or simplified with terms collected) for one remainder
M1: Attempts other remainder and puts one equal to the other
A1: for correct equation in a (and b) then **A1 for** $a = -20$ **cso**

Alternative for (a)

Uses long division, to get remainders as $b + 2a + 56$ or $b – a – 4$ or correct equivalent	M1 A1	
Uses second long division as far as remainder term, to get $b + 2a + 56 = b – a – 4$ or correct equivalent	M1 A1	
$a = – 20$	A1cso	5

Alternatives

M1: Uses long division of $x^4 + 5x^3 + ax + b$ by $(x \pm 2)$ **or** by $(x \pm 1)$ as far as three term quotient
A1: Obtains at least one correct remainder
M1: Obtains second remainder and puts two remainders (no x terms) equal
A1: correct equation **A1**: correct answer $a = –20$ following correct work.

(b)	$f(– 3) = (– 3)^4 + 5(– 3)^3 – 3a + b = 0$	M1 A1ft	
	$81 – 135 + 60 + b = 0$ gives $b = – 6$	A1 cso	3

<u>Notes</u>

M1: Puts $f(\pm 3) = 0$

A1 is for $f(– 3) = 0$, (where f is original function), with no sign or substitution errors (follow through on ‘a’ and could still be in terms of a)

A1: $b = – 6$ is cso.

Alternative for (b)

Uses long division of $x^4 + 5x^3 – 20x + b$ by $(x + 3)$ to obtain $x^3 + 2x^2 – 6x + a + 18$(with their value for a)	M1 A1ft	
Giving remainder $b + 6 = 0$ and so $b = – 6$	A1 cso	

Alternatives

M1: complete long division as far as constant (ignore remainder)

A1ft: needs correct answer for their a
A1: correct answer

Beware:

It is possible to get correct **answers with wrong working**. If remainders are equated to 0 in part (a) both correct answers are obtained fortuitously. This could score M1A1M0A0A0M1A1A0

[8]

5. (a) Attempt to find f(–4) or f(4). ($f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20$) M1
(= –128 – 48 + 156 + 20) = 0, so (x + 4) is a factor. A1 2

Long division scores no marks in part (a). The <u>factor theorem</u> is required.
However, the first two marks in (b) can be earned from division seen in (a) but if a different long division result is seen in (b), the work seen in (b) takes precedence for marks in (b).

A1 requires zero and a simple <u>conclusion</u> (even just a tick, or Q.E.D.), or may be scored by a <u>preamble</u>, e.g. 'If f(–4) = 0, (x + 4) is a factor…..'

(b) $2x^3 - 3x^2 - 39x + 20 = (x + 4)(2x^2 - 11x + 5)$ M1A1
.....$(2x - 1)(x - 5)$ (The 3 brackets need not be written together) M1A1cso 4
or ....$\left(x - \frac{1}{2}\right)(2x - 10)$ or equivalent

First M requires use of (x + 4) to obtain ($2x^2 + ax + b$), $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen… this could be done 'by inspection'.
Second M for the attempt to factorise their three-term quadratic.
Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $|cd| = |b|$ and $|pq| = |k|$.
If 'solutions' appear before or after factorisation, ignore…
… but factors must be seen to score the second M mark.

<u>Alternative (first 2 marks):</u>
$(x + 4)(2x^2 + ax + b) = 2x^3 + (8 + a)x^2 + (4a + b)x + 4b = 0$,
then compare coefficients to find <u>values</u> of a and b. [M1]
$a = -11, b = 5$ [A1]

<u>Alternative:</u>

Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0$.

$\therefore$ factor is, $(2x - 1)$ [M1, A1]
Finding that f (5) = 0 $\therefore$ factor is, $(x - 5)$ [M1, A1]

"Combining" all 3 factors is <u>not</u> required. If just one of these is found, score the <u>first 2 marks</u> M1A1M0A0

<u>Losing a factor of 2:</u> $(x + 4)\left(x - \frac{1}{2}\right)(x - 5)$ scores M1 A1 M1 A0

<u>Answer only, one sign wrong:</u>
e.g.$(x + 4)(2x - 1)(x + 5)$ scores M1 A1 M1 A0

[6]

6. (a) (i) $f(3) = 3^3 - 2 \times 3^2 - 4 \times 3 + 8; = 5$ M1; A1

(ii) $f(-2) = (-8 - 8 + 8 + 8) = 0$ (B1 on Epen, but A1 in fact) B1 3

M1 is for attempt at **either** f(3) or f(–3) in (i)
or f(–2) or f(2) in (ii).

No working seen: Both answers correct scores full marks
One correct ;M1 then A1B0 or A0B1,
whichever appropriate.

<u>Alternative (Long division)</u>
Divide by $(x - 3)$ OR $(x + 2)$ to get $x^2 + ax + b$, a may be zero [M1]
$x^2 + x - 1$ and + 5 seen i.s.w. (or "remainder = 5") [A1]
$x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1]

(b) $[(x + 2)](x^2 - 4x + 4)$ (= 0 not required) [must be seen or used in (b)] M1A1
$(x + 2)(x - 2)^2$ (= 0) (can imply previous 2 marks) M1
Solutions: × = 2 or – 2 (both) or (–2, 2, 2) [**no wrong working seen**] A1 4

First M1 requires division by a **found factor** ; e.g $(x + 2)$, $(x - 2)$ or
what candidate thinks is a **factor** to get $(x^2 + ax + b)$, a may be zero.
First A1 for $[(x + 2)]\ (x^2 - 4x + 4)$ or $(x - 2)(x^2 - 4)$
Second M1:attempt to factorise **their** found quadratic.
(or use formula correctly)
[Usual rule: $x^2 + ax + b = (x + c)(x + d)$, *where* $|cd| = |b|$.]

N.B. Second A1 is for solutions, not factors

SC: (i) Answers only:
Both correct, and no wrong, award M0A1M0A1 (as if B1,B1)
One correct, (even if 3 different answers) award M0A1M0A0 (as if B1)
(ii) Factor theorem used to find two correct factors, award
M1A1, then M0, A1 if both correct solutions given.
(–2, 2, 2 would earn all marks)
(iii) If in (a) candidate has $(x + 2)(x^2 - 4)$ B0, but then repeats
in (b), can score M1A0M1(if goes on to factorise)A0
(answers fortuitous)

<u>Alternative (first two marks)</u>

$(x + 2)(x^2 + bx + c) = x^3 + (2 + b)x^2 + (2b + c)x + 2c = 0$ and then
compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find b and c. [M1]
$b = -4$, $c = 4$ [A1]

<u>Method of grouping</u>

$x^3 - 2x^2 - 4x + 8 = x^2(x - 2)$,$4(x \pm 2)$ M1; $= x^2(x - 2) - 4(x - 2)$ A1
$[= (x^2 - 4)(x - 2)] = (x + 2)(x - 2)^2$ M1
Solutions: $x = 2$, $x = -2$ both A1

[7]

7. (a) $f(2) = 24 - 20 - 32 + 12 = -16$ (M: Attempt f(2) or f(–2)) M1A1 2

(If continues to say 'remainder = 16', isw)

Answer must be seen in part (a), not part (b).

Answer only (if correct) scores both marks. (16 as 'answer only' is M0A0).

<u>Alternative (long division):</u>

Divide by $(x - 2)$ to get $(3x^2 + ax + b)$, $a \neq 0$, $b \neq 0$. [M1]

$(3x^2 + x - 14)$, and –16 sen. [A1]

(if continues to say 'remainder = 16',isw)

(b) $(x + 2)(3x^2 - 11x + 6)$ M1A1

$(x + 2)(3x - 2)(x - 3)$ M1A1 4

(If continues to 'solve an equation', isw)

First M requires division by $(x + 2)$ to get $(3x^2 + ax + b)$, $a \neq 0$, $b \neq 0$).

Second M for attempt to factorise <u>their</u> quadratic, even if wrongly obtained, perhaps with a remainder from their division.

Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $|pq| = |k|$ and $|cd| = |b|$.

Just solving their quadratic by the formula is M0.

"Combining" all 3 factors is <u>not</u> required.

<u>Alternative (first 2 marks):</u>

$(x + 2)(3x^2 + ax + b) = 3x^3 + (6 + a)x^2 + (2a + b)x + 2b = 0$,

then compare coefficients to find <u>values</u> of a and b. [M1]

$a = -11$, $b = 6$ [A1]

<u>Alternative:</u>

Factor theorem: Finding that $f(3) = 0 \therefore$ factor is, $(x - 3)$ [M1, A1]

Finding that $f\left(\frac{2}{3}\right) = 0 \therefore$ factor is, $(3x - 2)$ [M1, A1]

If just one of these is found, score the <u>first 2 marks</u> M1A1M0A0.

<u>Losing a factor of 3:</u> $(x + 2)\left(x - \frac{2}{3}\right)(x - 3)$ scores M1A1M1A0.

<u>Answer only, one sign wrong:</u> e.g. $(x + 2)(3x - 2)(x + 3)$ scores M1A1M1A0.

[6]

8.	(a)	$f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$ $\{= -8 + 16 - 2 - 6\}$ $= 0$, $\therefore$ $x + 2$ is a factor	M1 A1	 2
		Attempting $f(\pm 2)$: No x s; allow invisible brackets for M mark Long division: M0 A0.	 M1	
		= 0 and minimal conclusion (e.g. factor, hence result, QED, ✓, □). If result is stated first [i.e. If $x + 2$ is a factor, $f(-2) = 0$] conclusion is not needed. Invisible brackets used as brackets can get M1 A1, so $f(-2) = -2^3 + 4 \times -2^2 + -2 - 6$ $\{ = -8 + 16 - 2 - 6\} = 0$, $\therefore x + 2$ is a factor M1 A1, but $f(-2) = -2^3 + 4 \times -2^2 + -2 - 6 = -8 - 16 - 2 - 6 = 0$, $\therefore x + 2$ is a factor M1 A0 Acceptable alternatives include: $x = -2$ is a factor, $f(-2)$ is a factor.	 A1	
	(b)	$x^3 + 4x^2 + x - 6 = (x + 2)(x^2 + 2x - 3)$ $= (x + 2)(x + 3)(x - 1)$	M1, A1 M1, A1	 4
		1st M1 requires division by $(x + 2)$ to get $x^2 + ax + b$ where $a \neq 0$ and $b \neq 0$ or equivalent with division by $(x + 3)$ or $(x - 1)$.	 M1	
		$(x + 2)(x^2 + 2x - 3)$ or $(x + 3)(x^2 + x - 2)$ or $(x - 1)(x^2 + 5x + 6)$ [If long division has been done in (a), minimum seen in (b) to get first M1 A1 is to make some reference to their quotient $x^2 + ax + b$.]	 A1	
		Attempt to factorise their quadratic (usual rules).	M1	
		"Combining" all 3 factors is not required.	A1	
		Answer only: Correct M1 A1 M1 A1 Answer only with one sign slip: $(x + 2)(x + 3)(x + 1)$ scores 1st M1 1st A1 2nd M0 2nd A0 $(x + 2)(x - 3)(x - 1)$ scores 1st M0 1st A0 2nd M1 2nd A1		
		Answer to (b) can be seen in (c).		
		Alternative comparing coefficients		
		$(x + 2)(x^2 + ax + b) = x^3 + (2 + a)x^2 + (2a + b)x + 2b$ Attempt to compare coefficients of two terms to find values of a and b	 M1	
		$a = 2$, $b = -3$	A1	
		Or $(x + 2)(ax^2 + bx + c) = ax^3 + (2a + b)x^2 + (2b + c)x + 2c$ Attempt to compare coefficients of three terms to find values of a, b and c.	 M1	
		$a = 1$, $b = 2$, $c = -3$	A1	
		Then apply scheme as above		

Alternative using factor theorem

Show f(–3) = 0; allow invisible brackets M1

$\therefore x + 3$ is a factor A1

Show f(1) = 0 M1

$\therefore x - 1$ is a factor A1

(c) –3, –2, 1 B1 1

–3, –2, 1 or (–3, 0), (–2, 0), (1, 0) only. Do not ignore subsequent working.
Ignore any working in previous parts of the question. Can be seen in (b) B1

Line in mark scheme in { } does not need to be seen.

[7]

9. (a) $f(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$ M: Attempt f(2) or f(–2) M1

$= -16 + 12 + 58 - 60 = -6$ A1 2

Alternative (long division):

Divide by (x + 2) to get ($2x^2 + ax + b$), $a \neq 0$, $b \neq 0$. [M1]

($2x^2 - x - 27$), remainder = –6 [A1]

(b) $f(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$ M: Attempt f(3) or f(–3) M1

$(= -54 + 27 + 87 - 60) = 0 \therefore (x + 3)$ is a factor A1 2

A1 requires zero and a simple conclusion (even just a tick, or Q.E.D.).

(c) $(x + 3)(2x^2 - 3x - 20)$ M1 A1

$= (x + 3)(2x + 5)(x - 4)$ M1 A1 4

First M requires division by (x + 3) to get ($2x^2 + ax + b$), $a \neq 0$, $b \neq 0$.

Second M for the attempt to factorise their quadratic.

Usual rule: ($2x^2 + ax + b$) = (2x + c)(x + d), where $|cd| = |b|$.

<u>Alternative (first 2 marks):</u>

$(x + 3)(2x^2 + ax + b) = 2x^3 + (6 + a)x^2 + (3a + b)x + 3b = 0$,
then compare coefficients to find <u>values</u> of a and b. [M1]
$a = –3, b = –20$ [A1]

<u>Alternative:</u>

Factor theorem:

Finding that $f\left(-\frac{5}{2}\right)=0$

∴ factor is, $(2x + 5)$ [M1, A1]

Finding that $f(4) = 0$

∴ factor is, $(x - 4)$ [M1, A1]

"Combining" all 3 factors is <u>not</u> required.
If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0.

<u>Losing a factor of 2</u>:

$(x + 3)\left(x+\frac{5}{2}\right)(x-4)$ scores M1 A1 M1 A0.

<u>Answer only, one sign wrong</u>:
e.g. $(x + 3)(2x - 5)(x - 4)$ scores M1 A1 M1 A0.

[8]

10. Uses f(2) = 0 to give	$16 - 4 + 2a + b = 0$	M1 A1		
Uses f(–1) = 6 to give	$-2 - 1 +-a + b = 6$	M1 A1		
Solves simultaneous equations to give	$a = -7$, and $b = 2$	M1 A1 A1	7	
				[7]

11. (a) $2 + 1 - 5 + c = 0$ or $-2 + c = 0$ — M1

$\underline{c = 2}$ — A1 — 2

(b) $f(x) = (x - 1)(2x^2 + 3x - 2)$ — $(x - 1)$ B1; division M1

$= \ldots\ \underline{(2x - 1)(x + 2)}$ — M1 A1 — 4

(c)	$f(\frac{3}{2}) = 2 \times \frac{27}{8} + \frac{9}{4} - \frac{15}{2} + c$		M1	
	Remainder = $c + 1.5$ = <u>3.5</u>	ft their c	A1ft	2

[8]

(a) M1 for evidence of substituting $x = 1$ leading to linear equation in c

(b) B1 for identifying $(x - 1)$ as a factor

1st M1 for attempting to divide.

Other factor must be at least $(2x^2$ + one other term)

2nd M1 for attempting to factorise a quadratic resulting from attempted division

A1 for just $(2x - 1)(x + 2)$.

(c) M1 for attempting $f(\pm\frac{3}{2})$. If not implied by $1.5 + c$, we must see some substitution of $\pm\frac{3}{2}$.

A1 follow through their c only, but it must be a number.

12. (a) Finding f (±2), and obtaining $16 - 32 + 10 + 6 = 0$ M1, A1 2

Or uses division and obtains $2x^2 - kx\ldots$, M1

obtaining $2x^2 - 4x - 3$ and concluding remainder = 0 A1

(b) Finding f $(\pm\frac{1}{2})$, and obtaining $-\frac{1}{4} - 2 - \frac{5}{2} + 6 = 1\frac{1}{4}$ M1,A1 2

Or uses division and obtains $x^2 - kx\ldots$,

obtaining $x^2 - \frac{9}{2}x + \frac{19}{4}$ and concluding remainder = $\frac{5}{4}$

-`

(c) $x = 2$ (also allow $\frac{2 \pm \sqrt{10}}{2}$ or $\frac{4 \pm \sqrt{40}}{4}$) B1 1

[5]

13.	(a)	Attempt to evaluate f(–4) ot f(4)	M1	
		$f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12 \quad (= 128 + 16 + 100 + 12) = 0$,		
		so…… is a factor	A1	2
	(b)	$(x + 4)(2x^2 - 7x + 3)$	M1 A1	
		………$(2x - 1)(x - 3)$	M1 A1	4
		First M requires ($2x^2 + ax + b$), $a \neq 0$, $b \neq 0$.		
		Second M for the attempt to factorise the quadratic.		

[6]

Alternative:

$(x + 4)(2x^2 + ax + b) = 2x^3 + (8 + a)x^2 + (4a + b)x + 4b = 0$, then compare coefficients to find values of a and b, — M1

$a = -7$, $b = 3$ — A1

Alternative:

Factor Theorem: Finding that $f\left(\frac{1}{2}\right) = 0$, $\therefore (2x - 1)$ is a factor — M1, A1

n.b. Finding that $f\left(\frac{1}{2}\right) = 0$, $\therefore (x - \frac{1}{2})$ is a factor scores M1, A0, unless the factor 2 subsequently appears.

Finding that $f(3) = 0$, $\therefore (x - 3)$ is a factor — M1, A1

14.	(a)	Attempt f (2) ; $= 16 - 4 + 4 - 16 = 0 \Rightarrow (x - 2)$ is a factor	M1; A1	
		must be statement for A1		
	(b)	$c = 8$	B1	2
		A complete method to find b		
		Either compare coefficients of x		
		or x^2: $-2b + 8 = 2$, or $-4 + b = -1$	M1	
		Or substitute value of x (may be implied):		
		e.g.$(x = 1) \Rightarrow -13 = (-1)(10 + b)$		
		$b = 3$	A1	3
	(c)	Checking $b^2 - 8c$; -55 $\Rightarrow$ no real roots to the quadratic	M1; A1	
		$\Rightarrow$ $x = 2$ is the only solution	A1	3

[8]

15.	(a)	$f(-2) = -16a - 4a + 6 + 7$	M1	
		$f(-2) = -3 \Rightarrow -20a + 13 = -3$		
		i.e. $20a = 16$	M1	
		$\underline{a = \frac{4}{5}}$ *o.e.*	A1	3
	(b)	$f\left(\frac{1}{2}\right) = \frac{a}{4} - \frac{a}{4} - \frac{3}{2} + 7 = \underline{\frac{11}{2}}$ *(o.e.)*	M1 A1	2
				[5]
16.	(a)	$f(2) = 1 \Rightarrow 8 - 2 \times 4 + 2a + b = 1$	M1 A1	
		$f(-1) = 28 \Rightarrow -1 - 2 - a + b = 28$	M1 A1	
		solving $\left.\begin{cases} 2a + b = 1 \\ -a + b = 31 \end{cases}\right\} \Rightarrow \underline{a = -10,\ b = 21}$	M1 A1	6
	(b)	$f(3) = 27 - 18 + 3a + b$	M1	
		$= 27 - 18 - 30 + 21 = 0$ $\quad \therefore (x - 3)$ is a factor	A1 c.s.o	2
				[8]
17.	(a)	$f(4) = 0 \Rightarrow 64 + 16(p + 1) - 72 + q = 0$	M1 A1	
		M1: f(4) or f(–4)		
		$16p + q + 8 = 0$ (*)	A1	3
	(b)	$f(-p) = 0 \Rightarrow -p^3 + p^2(p + 1) + 18p + q = 0$	M1 A1	
		M1: f(–p) or f(p)		
		$P^2 + 18p - q = 0$ (*)	A1	3
	(c)	Combine to form a quadratic equation in one unknown.	M1	
		$p^2 + 18p - (16p + 8) = 0$ $\quad p^2 + 2p - 8 = 0$	A1	
		$(p + 4)(p - 2) = 0$ $\quad p = \ldots,$ $\quad 2$	M1, A1cso	
		$q = -40$ $\quad$ (ft only for a positive p)	B1ft	5
	(d)	Complete method to find third factor.	M1	
		$x^3 + 3x - 18x - 40 = (x - 4)(x + 2)(x + 5)$	A1	2
				[13]

18.	(a)	<u>3</u>	B1	1
	(b)	$f(2) = 24 \Rightarrow 24 = (4 + p) \times 7 + 3$ Attempt $f(\pm 2)$	M1	
		<u>$\Rightarrow p = -1$</u> (*)	A1 cso	2
	(c)	$f(x) = (x^2 - 1)(2x + 3) + 3$ Attempt to multiply out	M1	
		$= 2x^3 + 3x^2 - 2x - 3 + 3$		
		$= x(2x^2 + 3x - 2)$ Factor of x	M1	
		= <u>$x(2x - 1)(x + 2)$</u> Attempt to factorise 3 term quadratic	M1, A1	4
				[7]

19.	(a)	$f(-2) = (-2)^3 - (19 \times -2) - 30$ M: Evaluate $f(-2)$ or $f(2)$	M1	
		$f(-2) = 0$, so $(x + 2)$ is a factor	A1	2
		<u>Alternative:</u> $(x^3 - 19x - 30) \div (x + 2) = (x^2 + ax + b)$, $a \neq 0$, $b \neq 0$ [M1]		
		$= (x^2 - 2x - 15)$, so $(x + 2)$ is a factor [A1]		
	(b)	$(x^3 - 19x - 30) = (x + 2)(x^2 - 2x - 15)$	M1 A1	
		$= (x + 2)(x + 3)(x - 5)$	M1 A1	4
				[6]

20.	(a)	Uses the remainder theorem with $x = ½$, or long division,		
		and puts remainder = 0	M1	
		To obtain $p + 2q = -35$ or any correct equivalent	A1	
		(allow more than 3 terms)		
		Uses the remainder theorem with $x = 1$, or long division,		
		and puts remainder = ± 7	M1	
		To obtain $p\ q = -21$ or any correct equivalent (allow more than 3 terms)	A1	
		Solves simultaneous equations to give $p = -7$, and $q = -14$	M1 A1	6
	(b)	Then $6x^3 - 7x^2 - 14x + 8 = (2x - 1)(3x^2 - 2x - 8)$	M1 A1 ft	
		So $f(x) = (2x - 1)(3x + 4)(x - 2)$	B1	3
				[9]

21.	(a)	Correct method for one of the 3 factors.	M1	
		$x(x - 1)(x - 5)$	M1 A1	3
		Allow $(x \pm 0)$ instead of x.		
		(2nd M1 for attempting full factorisation)		
	(b)	1 and 5	B1 ft	1

(c) $\frac{dy}{dx} = 3x^2 - 12x + 5$ M1 A1

At $x = 1$. $\frac{dy}{dx} = 3 - 12 + 5 = -4$ A1 3

(d) $\int (x^3 - 6x^2 + 5x)dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$ M1 A1

Evaluating at one of their x value: $\frac{1}{4} - 2 + \frac{5}{2}\left(= \frac{3}{4}\right)$ M1 A1 ft

Evaluating at the other x value: $\frac{625}{4} - 250 + \frac{125}{2}\left(= -31\frac{1}{4}\right)$ A1

$[\ldots..]_5 - [\ldots..]_1$ or $[\ldots..]_1 - [\ldots..]_5$ M1

$$-31\frac{1}{4} - \frac{3}{4} = -32$$

Total Area = $32 + \frac{3}{4} = 32\frac{3}{4}$ A1 7

If integrating the wrong expression in (d), (e.g. $x^2 - 6x + 5$), do not allow the first M mark, but then follow scheme.

[14]

22. (a) $p + 6 + 12 + q = -\frac{1}{8}p + \frac{6}{4} - 6 + q$ M1, M1

$\therefore \frac{9}{8}p = -22\frac{1}{2}$ M1

$p = -20$ A1 cso 4

(b) Remainder = $p + q + 18 = p + 21$ (=1) or $-\frac{p}{8} - \frac{3}{2}$ B1 ft on p 1

[5]

23. (a) Using f(±2) = 3 M1

Showing that p = 6 ✸, with no wrong working seen. A1 2

S.C. If p = 6 used and the remainder is shown to be 3 award B1

(b)	Attempt to find quotient when dividing $(n + 2)$ into $f(n)$ or attempting to equate coefficients.	M1		
	Quotient = $n^2 + 4n + 3$, or finding either $q = 1$ or $r = 3$	A1		
	Finding both $q = 1$ and $r = 3$	A1	3	
(c)	The product of three consecutive numbers must be divisible by 3	M1		
	Complete argument	A1	2	
				[7]

24.	$(x-1)$:	subst $x = 1$	$a + 3 + b + 1 = 5$	M1 A1		
		subst $x = -2$	$-8a + 12 - 2b + 1 = -1$			
			$a + b = 1$	A1		
			$8a + 2b = 14$			
			$a = 2, \quad b = -1$	M1 A1	5	
						[5]

25.	(a)	$f(1) = 0, \quad 2 - 1 + p + 6 = 0 \quad$ so $p = -7$	M1 A1	2	
	(b)	$f(-\frac{1}{2}) = -\frac{1}{4} - \frac{1}{4} + \frac{7}{2} + 6 = 9$	M1 A1	2	
					[4]

1. Although many candidates opted for long division rather than the remainder theorem in part (a), most scored the method mark and many accurately achieved the correct value for the remainder.

Long division in part (b) often led to the correct quadratic, which most candidates factorised correctly. Correct factorisation by inspection was seen occasionally, but attempting (by trial and error) to find further solutions by using the factor theorem was rarely successful. Some candidates, having found factors, thought they had finished and did not proceed to give any solutions to the equation. The 'obvious' solution $x = 5$ was sometimes omitted. 'Implicit' solutions such as $f(5) = 0$ were generously allowed on this occasion.

2. (a) Most who used the remainder theorem correctly used $f(\frac{1}{2})$ and equated it to –5, then used f(–2) and equated it to zero. They then solved simultaneous equations. There were a number of errors simplifying fractions and dealing with negative numbers and so a significant minority of the candidates scored the three method marks but lost all three accuracy marks. Some candidates forgot to equate their first expression to –5 and some wrote expressions not equations. There were also a number of errors rearranging terms and dealing with fractions. A small minority thought that $a(\frac{1}{2})^2$ became $\frac{1}{4}a^2$. It was obvious from the multiple efforts and crossings-out that a number of candidates were unhappy with their a and b values, but were often unable to resolve their problems. Those who used long division very rarely got as far as a correct remainder. They usually made little progress, and penalised themselves by the excessive time taken to do the complicated algebra required.

(b) Most candidates attempted this part of the question, even after limited success in part (a). It was common for those candidates who found fractional values for a or b to multiply f(x) by a denominator to create integer coefficients here. Division by $(x + 2)$ was generally done well using "long division" or synthetic division and candidates who had achieved full marks in part (a) normally went on to achieve full marks in (b), with the common error being failing to factorise their quadratic expression correctly. A significant group stopped at the quadratic factor and so lost the final two marks.

Candidates completing this question successfully were careful and accurate candidates and the question proved discriminating. A number of candidates made several attempts, sometimes achieving success on the third try.

3. The style of this question on the remainder theorem was unusual and candidates' performance was generally disappointing. In part (a), finding the value of f (k) proved surprisingly difficult. Many candidates seemed unable to appreciate that $(3k – 2)(k – k) – 8$ could be simplified to –8, and $3k$ –10 was a popular answer.

Thankfully the majority attempted to use the remainder theorem rather than long division (which was very rarely successful) in part (b), but numerical and algebraic mistakes were very common. Sometimes the expression for the remainder was equated to 0 rather than 4, losing the method mark.

Some candidates had no idea of how to proceed in part (c) and those who made progress were often unable to reach the correct factorised form of the resulting quadratic expression. Some solved a quadratic equation by use of the formula at this stage, never achieving the required factorised form.

4. In part (a) most who used the remainder theorem correctly used f(2) and f(–1) and scored M1A1 usually for 16 + 40 + 2a + b, the (–1)^4 often causing problems. A large number of candidates then mistakenly equated each to zero and solved the equations simultaneously, obtaining a = –20 and ignoring b = –16 so that they could go on in (b) to use f(–3) = 0 to obtain b = –6.

Those who equated f(2) to f(–1), as required, usually completed to find a although there were many careless errors here. Some candidates worked with f(2) – f(–1) and then equated to zero but not always very clearly.

The candidates using long division often made a small error, which denied most of the marks available:

- Omission of the "$0x^2$" term as a place-holder from the dividend resulted in much confusion.
- Failure to pursue the division until they had reached the constant term gave equations of the "remainders" still containing x.
- The almost inevitable habit of subtracting negative terms wrongly (e.g., 5 × 2 – (–2 × 2) = 3 × 2).

They usually made little progress, and penalised themselves by the excessive time taken to do the complicated algebra required.

In part (b) again the remainder theorem method scored better than the long division method. Most candidates who reached a = –20 obtained the correct value for b, but there was some poor algebra, with the powers of –3 causing problems for some. A few used f(3) instead of f(–3) and a number did not set their evaluation equal to zero.

5. Part (a) of this question required the use of the factor theorem (rather than long division) and most candidates were able to show f(–4) = 0. As in previous papers, a simple conclusion was expected. Many candidates failed to provide this.
The most popular strategy in part (b) was to use long division, dividing the cubic expression by (x + 4) to find the quadratic factor. Some candidates stopped at that stage and so could only gain a maximum of two marks, but of those who reached $2x^2 - 11x + 5$ and went on to factorise this, the vast majority gained full marks. Less formal approaches to the division, including 'division by inspection', were occasionally seen and usually effective.

Candidates who solved $2x^2 - 11x + 5 = 0$ gained neither of the final two marks until they produced the relevant factors, and then one of the factors was often left as $\left(x - \frac{1}{2}\right)$, which lost the final mark unless the factor 2 was included.

Some candidates went on to give 'solutions' $x = -4$, $x = 5$, $x = \frac{1}{2}$, suggesting confusion over the meaning of 'factorise'.

6. The fact that $(x + 2)$ and $(x – 2)$ were both factors of the cubic was unfortunate and examiners needed to be eagle-eyed in marking part (a); some candidates clearly evaluated f(2) in answering (a)(ii). There were often arithmetic errors in evaluating f(–2), with 16 being a common answer, and consequently many candidates had not found a factor in (a) and needed to start from scratch in part (b).

Of those candidates who chose to use long division in (a), there was a considerable number who produced $(x + 2)(x^2 – 4)$ in (ii) and then went on to use this in part (b). Although the solutions $x = 2$ and $x = –2$ were then often still found, this was fortuitous and M1A0M1A0 was a common outcome. The most frequent loss of the final mark, however, was for giving the factors, not the solutions, to the cubic equation.

7. In part (a), many candidates unnecessarily used long division rather than the remainder theorem to find the remainder. The correct remainder –16 was often achieved, although mistakes in arithmetic or algebra were common.
There were many good solutions to the factorisation in part (b). Candidates usually found the quadratic factor by long division or by 'inspection' and went on to factorise this quadratic, obtaining the correct linear factors. Sometimes time was wasted in justifying the given fact that $(x + 2)$ was a factor. Some candidates were distracted by part (a) and assumed that $(x – 2)$ was one of the factors, using the quadratic they had obtained from their long division in part (a).
A few attempted to use the formula to find the roots of the quadratic but did not always continue to find the factors. It was common for solutions of the equation $f(x) = 0$ to be given, but this 'additional working' was not penalised here.

8. Many candidates gained full marks for this question. Candidates who attempted long division in part (a) rather than using the factor theorem lost both marks and those who showed that f(–2) = 0 but failed to give a conclusion lost the accuracy mark. Parts (b) and (c) were usually answered successfully although some candidates showed a lack of understanding of the difference between factorising and solving. The majority of candidates used long division in (b) rather than inspection. Some lost the final mark in part (c) by giving only two solutions (usually –3 and 1) rather than three.

9. Many candidates unnecessarily used long division in part (a) to find the remainder. The correct remainder –6 was often achieved, but sometimes the answer 6 followed correct working. Careless algebraic and arithmetic mistakes spoilt some solutions. Candidates who used long division rather than the factor theorem lost the marks in part (b) of this question, and those who obtained f(–3) = 0 but failed to give a conclusion lost the second mark.
There were many good solutions to the factorisation in part (c). Candidates usually found the quadratic factor by long division (which was generally well understood) or by 'inspection' and went on to factorise this quadratic, obtaining the correct linear factors. Some of the weaker candidates failed to recognise that $(x + 3)$ from part (b) was one of the factors and tried to use $(x + 2)$ from part (a). A few attempted to use the formula to find the roots of the quadratic and then to use the roots to find the factors. This was not always successful, as it tended to lead to the loss of a factor of 2 in the final answer.

10. The vast majority of candidates scored full marks for this question. A few incorrectly stated that f(1) = 6 or f(–1) = 0 but were still able to gain some marks by using a valid method to solve simultaneous equations in a and b. The most common error was seen in evaluating f(–1) when $-(-1)^2$ was given as +1.

11. This proved to be a friendly starter question for most candidates and it was usually answered correctly. Part (a) caused few problems and most candidates used the factor theorem as intended. Most realized that $(x - 1)$ was a factor in part (b) and proceeded with some division. In the majority of cases this was completed correctly and the resulting quadratic factor was factorized successfully too. Some candidates do not appreciate the difference between “factorize” and “solve”. Some used a quadratic formula to find the roots of their quadratic factor, occasionally they then tried to turn these roots into factors but invariably lost the 2 from $(2x - 1)$. Others went on from a correct factorization to solve $f(x) = 0$, but there was no penalty for this on this occasion. In part (c) a number of students used division, rather than the remainder theorem. This wasted time and created more opportunities for errors but well over half of the candidates found the correct value for the remainder.

12. Most candidates attempted parts a) and b) successfully, with the vast majority using the remainder theorem, and very few using long division. Many of them did not do part c) correctly. There seemed to be some confusion between the *factor* $(x - 2)$ and the *solution* x=2. A number of candidates found all the solutions of the equation doing an unnecessary amount of work for the one mark available.

13. Candidates who used long division rather than the factor theorem lost the marks in part (a) of this question, and those who obtained f(–4) = 0 but failed to give a conclusion lost the second mark.

There were many good solutions to the factorisation in part (b). Candidates usually found the quadratic factor by long division (which was generally well understood) or by ‘inspection’ and went on to factorise this quadratic, obtaining the correct linear factors. Occasionally the factor theorem was used to establish one or both of the remaining linear factors. Having found linear factors, it was tempting for some candidates to solve a non-existent equation, but examiners ignored such ‘subsequent working’. Those candidates whose first step was $x(2x^2 + x - 25) + 12$ made no progress.

14. (a) Generally this was done very well with most candidates who used the factor theorem getting both marks. A few didn't get the A1 for giving a statement, and a few also used long division instead of the factor theorem so had no marks at all.

(b) The majority of candidates used long division , and mistakes, if made, were on the subtraction within the division (b=-5 was regularly seen). A number of candidates multiplied out and compared coefficients.

(c) Full marks was common, but those who lost marks seemed to be reluctant on a maths paper to write words! Conclusions were often missing or incomplete, and mistakes were made with the definitions of factor and solution – statements such as the solution or root is x-2 , or x=2 is a factor were seen often.

15. This proved to be a comfortable starter question with most candidates who used the remainder theorem scoring full marks. Those choosing to long divide ran into more difficulties but usually managed to complete both parts. A common error was to use f(½) = +3 in part (a); however the question did not penalise candidates in part (b).

16. The remainder theorem was the favoured (and intended) approach here and there were many perfectly correct solutions to this question using, only a small minority losing marks due to algebraic slips. Those who tried to use long division were usually less successful in obtaining two correct equations. Part (b) is a "show that" question and therefore requires some comment from the candidates in order to secure full marks, some merely showed f(3)=0 and therefore lost the final mark.

17. Although parts of the algebra in this question were quite demanding, an encouraging number of candidates were able to produce excellent solutions. The factor theorem was often used accurately in parts (a) and (b), but f $(-p)$ was not always well handled, and some candidates did not make it clear that f (4) and f $(-p)$ were respectively equal to zero. Just a few used other methods such as long division in their attempts to answer parts (a) and (b), but these candidates rarely made any significant progress.

In part (c), solutions to the simultaneous equations were usually completed correctly, although those who found an equation in q rather than p made it much more difficult for themselves. Sometimes the equation in p was achieved in a somewhat dubious fashion by simplifying $p^2 + 18p + q = 16p + q + 8$.

Despite knowing two factors of f (x) from the earlier parts of the question, many candidates were not able to complete the full factorisation in part (d). Some had no idea how to start, but for those who made progress the most popular approach was to divide the cubic by $(x - 4)$, then to factorise the resulting quadratic.

18. This question was answered well with almost all candidates scoring full marks in part (b). In part (a) many candidates preferred to evaluate f$(-\frac{3}{2})$ rather than simply writing down the answer. In some cases there were errors in the arithmetic and the remainder was an expression in *p*. In part (c) some candidates did not multiply out the brackets and simplify. Instead they just factorized ($x^2 - 1$). Others failed to spot the factor of *x* and used the remainder theorem along with long division to complete the factorization, whilst a few were convinced that (x – 2) was a factor and abandoned the question when difficulties arose. Those candidates with good algebraic skills moved quickly to a correct answer in a couple of lines.

19. Most candidates were able to make progress in this question. In part (a), those who used the factor theorem rather than division methods were usually more successful, but it was common for a conclusion to be omitted. Having found a quadratic factor either in part (a) or part (b), the majority continued to find the correct complete factorised form. Attempts to use the quadratic formula on the given cubic function were occasionally seen. Some candidates showed a lack of understanding of "factor", indicated by statements such as "–2 is a factor of f(x)", and others, having factorised, were keen to continue to solve an equation. "Subsequent working" of this kind was not penalised.

20. Most of the candidates successfully used factor and remainder theorems, and there were fortunately few attempts at long division. The two simultaneous equations were usually solved well, and most made an attempt at the final factorisation. The final mark was sometimes lost for the factor $(x + \frac{4}{3})$ rather than ($3x$ +4).

21. High marks were often scored in this question. The factorisation in part (a) proved surprisingly difficult for some candidates, especially those who failed to use *x* as a factor. Some used the factor theorem to show that (x – 1) was a factor, and then used long division, but failed to factorise $(x^2 - 5x)$. Even these, however, usually realised that 1 and 5 were the required *x*-coordinates in part (b). While most candidates found the gradient correctly in part (c), it was notable that others failed to realise that differentiation was needed.

Apart from arithmetic slips, the majority of candidates were able to integrate and substitute limits correctly in part (d), where the only real problem was in dealing with the negative value (region below the *x*-axis) for the integral from 1 to 5. Here, some tried to compensate for the negative value in unusual ways and never managed to reach an appropriate answer for the combined area.

22. The great majority of candidates were familiar with the remainder theorem. There were a few errors due to substituting +½ rather that -½, but most scored the first two marks. Some candidates put both remainders equal to zero and solved simultaneous equations. Very few candidates attempted to do part (a) by division. Even those who went wrong in part (a) often managed to get the follow thorough mark in part (b). This question was answered well by most candidates.

23. Most were able to use the remainder theorem correctly and thus showed that $p = 6$. Part (b) was also answered well with the majority of candidates obtaining full marks. The essential argument required in their answer to part (c) was that for *any positive integer n*, one of the brackets would be a multiple of 3 and therefore the product would also be a multiple of 3, and the sum of a multiple of 3 and 3 itself would also be a multiple of 3.

24. No Report available for this question.

25. No Report available for this question.