

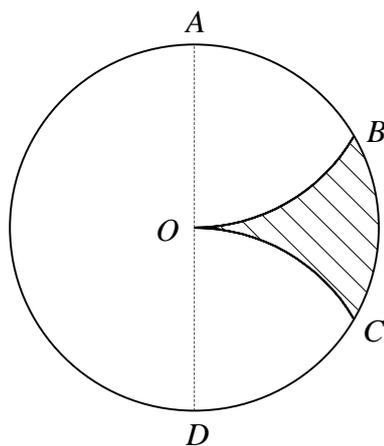
Core Mathematics C2 Paper L

1. (i) Sketch on the same diagram the graphs of $y = \sin 2x$ and $y = \tan \frac{x}{2}$ for x in the interval $0 \leq x \leq 360^\circ$. [4]
- (ii) Hence state how many solutions exist to the equation

$$\sin 2x = \tan \frac{x}{2},$$

for x in the interval $0 \leq x \leq 360^\circ$ and give a reason for your answer. [2]

2.



The diagram shows a circle of radius r and centre O in which AD is a diameter.

The points B and C lie on the circle such that OB and OC are arcs of circles of radius r with centres A and D respectively.

Show that the area of the shaded region OBC is $\frac{1}{6}r^2(3\sqrt{3} - \pi)$. [6]

3. The sequence u_1, u_2, u_3, \dots is defined by

$$u_{n+1} = (u_n)^2 - 1, \quad n \geq 1.$$

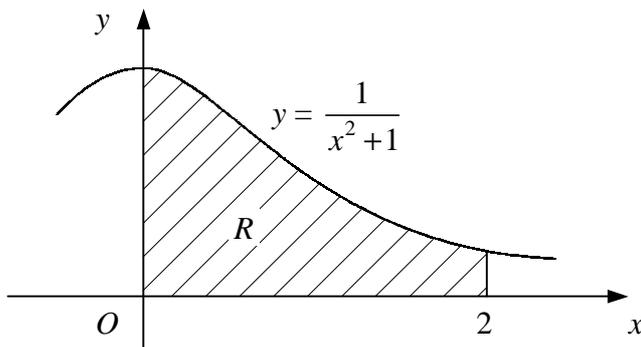
Given that $u_1 = k$, where k is a constant,

- (i) find expressions for u_2 and u_3 in terms of k . [3]

Given also that $u_2 + u_3 = 11$,

- (ii) find the possible values of k . [4]

4.



The diagram shows the curve with equation $y = \frac{1}{x^2 + 1}$.

The shaded region R is bounded by the curve, the coordinate axes and the line $x = 2$.

(i) Use the trapezium rule with four strips of equal width to estimate the area of R . [5]

The cross-section of a support for a bookshelf is modelled by R with 1 unit on each axis representing 8 cm. Given that the support is 2 cm thick,

(ii) find an estimate for the volume of the support. [2]

5. (i) Find the value of a such that

$$\log_a 27 = 3 + \log_a 8. \quad [3]$$

(ii) Solve the equation

$$2^{x+3} = 6^{x-1},$$

giving your answer to 3 significant figures. [4]

6. (i) Evaluate

$$\int_2^4 \left(2 - \frac{1}{x^2}\right) dx. \quad [4]$$

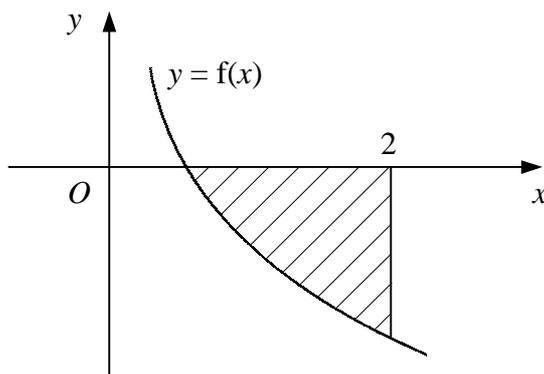
(ii) Given that

$$\frac{dy}{dx} = 2x^3 + 1,$$

and that $y = 3$ when $x = 0$, find the value of y when $x = 2$. [5]

Turn over

7.



The diagram shows part of the curve $y = f(x)$ where $f(x) = \frac{1-8x^3}{x^2}$, $x \neq 0$.

- (i) Solve the equation $f(x) = 0$. [3]
- (ii) Find $\int f(x) dx$. [3]
- (iii) Find the area of the shaded region bounded by the curve $y = f(x)$, the x -axis and the line $x = 2$. [3]

8. A store begins to stock a new range of DVD players and achieves sales of £1500 of these products during the first month. In a model it is assumed that sales will decrease by £ x in each subsequent month, forming an arithmetic sequence.

Given that sales total £8100 during the first six months, use the model to

- (i) find the value of x , [4]
- (ii) find the expected value of sales in the eighth month, [2]
- (iii) show that the expected total of sales in pounds during the first n months is given by $kn(51 - n)$, where k is an integer to be found. [3]
- (iv) Explain why this model cannot be valid over a long period of time. [1]

9.

$$f(x) = 2x^3 - 5x^2 + x + 2.$$

- (i) Show that $(x - 2)$ is a factor of $f(x)$. [2]
- (ii) Fully factorise $f(x)$. [4]
- (iii) Solve the equation $f(x) = 0$. [1]
- (iv) Find, in terms of π , the values of θ in the interval $0 \leq \theta \leq 2\pi$ for which

$$2 \sin^3 \theta - 5 \sin^2 \theta + \sin \theta + 2 = 0. [4]$$