S14 C2(R)

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1. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1+\frac{3x}{2}\right)^8$$

giving each term in its simplest form.

(4)

$$(a+b)^8 = a^8 + 8a^7b + \binom{8}{2}a^6b^2 + \binom{8}{3}a^5b^3$$

$$\left(1+\frac{3x}{2}\right)^8 = 1 + 8\left(\frac{3x}{2}\right) + 28\left(\frac{3x}{2}\right)^2 + 56\left(\frac{3x}{2}\right)^3$$

$$= 1 + 12x + 63x^2 + 189x^3$$

2. A geometric series has first term a, where $a \neq 0$, and common ratio r.
The sum to infinity of this series is 6 times the first term of the series.

(a) Show that $r = \frac{5}{6}$

(2)

Given that the fourth term of this series is 62.5

(b) find the value of a.

(2)

(c) find the difference between the sum to infinity and the sum of the first 30 terms, giving your answer to 3 significant figures.

(4)

a) $S_\infty = \frac{a}{1-r} = 6a \Rightarrow \frac{a}{6a} = 1-r$

$\Rightarrow \frac{1}{6} = 1-r$ $\times 6$ $1 = 6 - 6r \Rightarrow 6r = 5 \therefore r = \frac{5}{6}$ *

b) $u_4 = ar^3 = a\left(\frac{5}{6}\right)^3 = 62.5 \therefore a = 108.$

c) $S_{30} = 108\frac{\left(1-\left(\frac{5}{6}\right)^{30}\right)}{\frac{1}{6}} = 645.27...$

$\therefore$ difference $= \frac{108}{\left(\frac{1}{6}\right)} - 645.27..$

$= 2.73$

3.

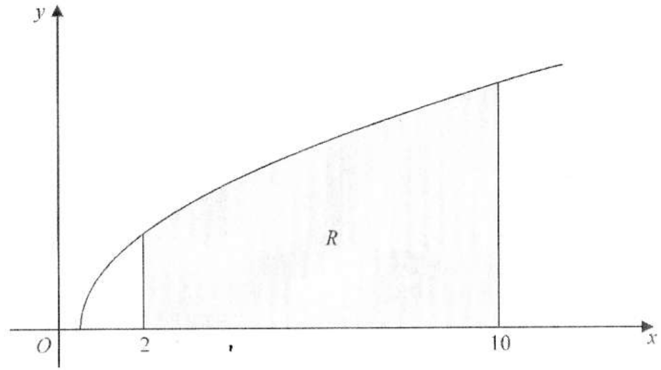

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(2x - 1)}$, $x \geqslant 0.5$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 10$.

The table below shows corresponding values of x and y for $y = \sqrt{(2x - 1)}$.

$h=2$

x	2	4	6	8	10
y	$\sqrt{3}$	$\sqrt{7}$	$\sqrt{11}$	$\sqrt{15}$	$\sqrt{19}$

(a) Complete the table with the values of y corresponding to $x = 4$ and $x = 8$. **(1)**

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places. **(3)**

(c) State whether your approximate value in part (b) is an overestimate or an underestimate for the area of R. **(1)**

Area $\approx \frac{1}{2}(2)[\sqrt{3}+2(\sqrt{7}+\sqrt{11}+\sqrt{15})+\sqrt{19}) = 25.76$

c) under estimate, trapezia will be below curve.

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4. $f(x) = -4x^3 + ax^2 + 9x - 18$, where a is a constant.

Given that $(x - 2)$ is a factor of $f(x)$,

(a) find the value of a, **(2)**

(b) factorise $f(x)$ completely, **(3)**

(c) find the remainder when $f(x)$ is divided by $(2x - 1)$. **(2)**

a) $f(2) = 0 \Rightarrow -32 + 4a + \cancel{18} - \cancel{18} = 0 \quad \therefore \quad a = 8$

b)

	$-4x^2$	$+9$	
x	$-4x^3$	$9x$	/
-2	$8x^2$	-18	/

$r=0$

$= -(x-2)(4x^2-9)$

$= -(x-2)(2x-3)(2x+3)$

c) $f\left(\frac{1}{2}\right) = -4\left(\frac{1}{8}\right) + 8\left(\frac{1}{4}\right) + 9\left(\frac{1}{2}\right) - 18 = -12$

$\therefore$ remainder is -12

5.

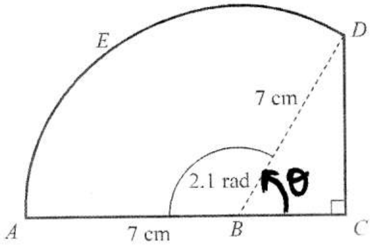

Figure 2

Figure 2 shows the shape $ABCDEA$ which consists of a right-angled triangle BCD joined to a sector $ABDEA$ of a circle with radius 7 cm and centre B.

A, B and C lie on a straight line with $AB = 7$ cm.

Given that the size of angle ABD is exactly 2.1 radians,

(a) find, in cm, the length of the arc DEA, **(2)**

(b) find, in cm, the perimeter of the shape $ABCDEA$, giving your answer to 1 decimal place. **(4)**

a) arc $= r\theta = 7 \times 2.1 = 14.7$

b) $\theta = \pi - 2.1$

$BC = 7\cos(\pi - 2.1) = 3.53$

$CD = 7\sin(\pi - 2.1) = 6.04$

$P = 31.3$

blank

6.

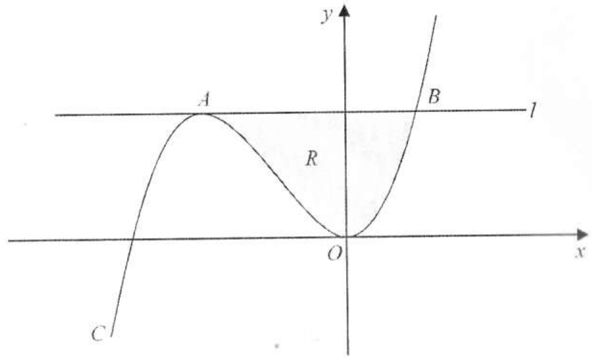

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{8}x^3 + \frac{3}{4}x^2, \quad x \in \mathbb{R}$$

The curve C has a maximum turning point at the point A and a minimum turning point at the origin O.

The line l touches the curve C at the point A and cuts the curve C at the point B.

The x coordinate of A is -4 and the x coordinate of B is 2.

The finite region R, shown shaded in Figure 3, is bounded by the curve C and the line l.

Use integration to find the area of the finite region R. **(7)**

$x = 2 \quad y = \frac{1}{8}(8) + \frac{3}{4}(4) = 4$

$\boxed{24}$ 4

-4 6 2

$\therefore R = 24 - \int_{-4}^{2} \frac{1}{8}x^3 + \frac{3}{4}x^2 \, dx$

$= 24 - \left[\frac{1}{32}x^4 + \frac{3}{12}x^3\right]_{-4}^{2}$

$= 24 - \left[\left(\frac{5}{2}\right) - (-8)\right] = 13.5$

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7. (i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place. **(3)**

(ii) Solve, for $0 \leqslant x < 2\pi$, the equation

$$5\sin^2 x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.
(Solutions based entirely on graphical or numerical methods are not acceptable.) **(5)**

i) $\sin 2\theta = 4\sin 2\theta - 1 \Rightarrow 3\sin 2\theta = 1$

$\Rightarrow \sin 2\theta = \frac{1}{3} \Rightarrow 2\theta = \sin^{-1}\left(\frac{1}{3}\right) = 0.3398$

$2\theta = 0.3398, 2.8018 \ldots$

$\theta = 0.2^c, 1.4^c \quad \therefore \theta = 9.7°, 80.3°$

ii) $5(1-\cos^2 x) - 2\cos x - 5 = 0$

$\Rightarrow 5\cos^2 x + 2\cos x = 0$

$\Rightarrow \cos x(5\cos x + 2) = 0$

$\cos x = 0 \qquad \cos x = -\frac{2}{5}$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = 1.98^c, 4.30^c$

$2\pi -$

8. (i) Solve

$$5^y = 8$$

giving your answer to 3 significant figures. **(2)**

(ii) Use algebra to find the values of x for which

$$\log_2(x + 15) - 4 = \frac{1}{2}\log_2 x$$

(6)

i) $y = \log_5 8 = 1.29$

ii) $\log_2(x+15) - \log_2(\sqrt{x}) = 4$

$\log_2\left(\frac{x+15}{\sqrt{x}}\right) = 4 \Rightarrow \frac{x+15}{\sqrt{x}} = 2^4 = 16$

$x + 15 = 16\sqrt{x}$

$x - 16\sqrt{x} + 15 = 0$

$(\sqrt{x} - 15)(\sqrt{x} - 1) = 0$

$\therefore \sqrt{x} = 15 \qquad \sqrt{x} = 1$

$x = 225 \qquad x = 1$

9.

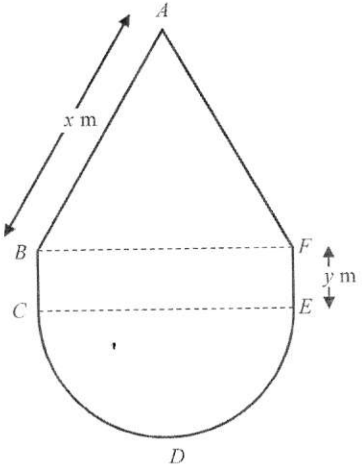


Figure 4

Figure 4 shows the plan of a pool.

The shape of the pool $ABCDEFA$ consists of a rectangle $BCEF$ joined to an equilateral triangle BFA and a semi-circle CDE, as shown in Figure 4.

Given that $AB = x$ metres, $EF = y$ metres, and the area of the pool is 50 m^2,

(a) show that

$$y = \frac{50}{x} - \frac{x}{8}(\pi + 2\sqrt{3})$$

(3)

(b) Hence show that the perimeter, P metres, of the pool is given by

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3})$$

(3)

(c) Use calculus to find the minimum value of P, giving your answer to 3 significant figures.

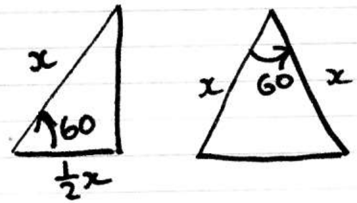
(5)

(d) Justify, by further differentiation, that the value of P that you have found is a minimum.

(2)

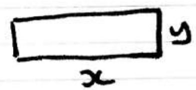
Question 9 continued

a)

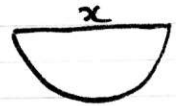


area $= \frac{1}{2}(x)(x)\sin 60$

$= \frac{\sqrt{3}}{4}x^2$



area $= xy$



Area $= \frac{\pi\left(\frac{1}{2}x\right)^2}{2} = \frac{1}{8}\pi x^2$

$xy + \frac{1}{8}\pi x^2 + \frac{\sqrt{3}}{4}x^2 = 50$

$xy = 50 - \frac{1}{8}x^2(\pi - 2\sqrt{3})$

$\therefore y = \frac{50}{x} - \frac{1}{8}x(\pi - 2\sqrt{3})$

b) $P = x + x + y + \frac{1}{2}\pi x + y$

$P = 2y + (2 + \frac{1}{2}\pi)x$

$P = \frac{100}{x} - \frac{x}{4}(\pi + 2\sqrt{3}) + (2 + \frac{1}{2}\pi)x$

$P = \frac{100}{x} + \frac{x}{4}(8 + 2\pi - \pi - 2\sqrt{3})$

$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3})$ *

$$P = 100x^{-1} + \left(\frac{\pi + 8 - 2\sqrt{3}}{4}\right)x$$

$$P' = -100x^{-2} + \left(\frac{\pi + 8 - 2\sqrt{3}}{4}\right)$$

$$P'' = 200x^{-3}$$

at Min $P' = 0 \Rightarrow \frac{100}{x^2} = \left(\frac{\pi + 8 - 2\sqrt{3}}{4}\right)$

$\Rightarrow x^2 = \frac{400}{\pi + 8 - 2\sqrt{3}} \quad \therefore x = 7.22$

$\therefore \text{Min } P = \frac{100}{7.22\ldots} + \left(\frac{\pi + 8 - 2\sqrt{3}}{4}\right)(7.22\ldots)$

$\text{min } P = 27.7$

d) at $x = 7.22 \quad P'' = \frac{200}{x^3} > 0$

$\therefore P$ is a minimum

10. The circle C, with centre A, passes through the point P with coordinates $(-9, 8)$ and the point Q with coordinates $(15, -10)$.

Given that PQ is a diameter of the circle C,

(a) find the coordinates of A.
(2)

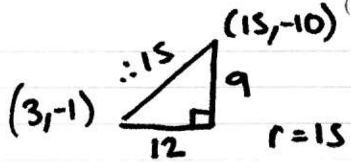
(b) find an equation for C.
(3)

A point R also lies on the circle C.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.
(2)

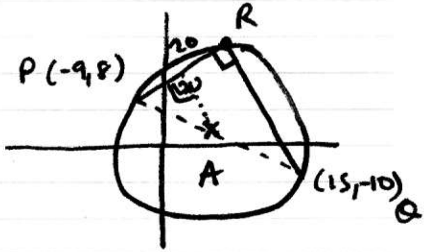
(d) Find the size of the angle ARQ, giving your answer to the nearest 0.1 of a degree.
(2)

a) centre A $(3, -1)$



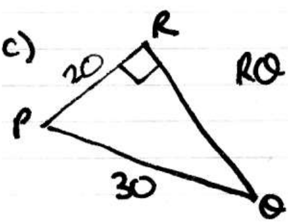
$r = 15$

b) $(x-3)^2 + (y+1)^2 = 225$



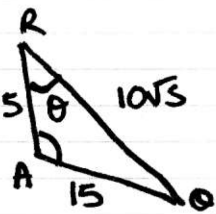
$\therefore \sqrt{15^2 - 10^2} = 5\sqrt{5}$

c)



$RQ = \sqrt{30^2 - 20^2}$

$= 10\sqrt{5}$



$\cos\theta = \dfrac{(10\sqrt{5})^2 + 15^2 - 15^2}{2 \times 15 \times 10\sqrt{5}}$ $\quad \theta = 41.8$