

CORE MATHEMATICS (C) UNIT 2 TEST PAPER 4

1. In the triangle ABC , $AB = 2\sqrt{2}$ cm, $AC = 3.5$ cm and angle $BAC = \frac{\pi}{4}$ radians.

Calculate the length of BC . [4]

2. Find the values of a and b for which $\sum_{r=0}^{n-1} (4r + 2) = \sum_{s=1}^n (as + b)$. [5]

3. Given that $2 \log_a x - \log_a 4 = 3$, where a is a positive constant,
 (i) express x in terms of a . [3]
 (ii) Find the value of x when $a = 9$. [2]

4. Find the first four terms, in ascending powers of x , in the binomial expansion of $(3 - 4x)^5$. [6]

5. The table gives some values of $\sqrt{\log_{10} x}$.

x	1	1.5	2	2.5	3	3.5
$\sqrt{\log_{10} x}$	0	0.420		0.631		0.738

- (i) Calculate, to 3 decimal places, the missing values in the table. [2]

- (ii) Using all six values, estimate the value of $\int_1^{3.5} \sqrt{\log_{10} x} dx$ to 2 decimal places. [4]

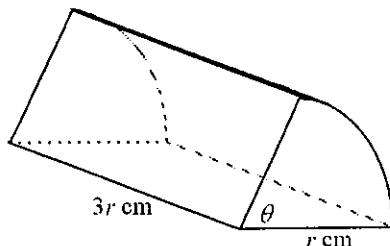
6. Find all the solutions in the interval $0 < x < 180$ of the equations
 (i) $\sin(x - 20)^\circ = \sin 80^\circ$, [3]
 (ii) $\tan 3x^\circ = -\sqrt{3}$. [4]

7. The first three terms of a geometric series are respectively $p, p^2, p^2 + 4$.
 (i) Show that the only possible real value of p is 2. [5]
 (ii) If the n th term of this series is T_n and the sum of the first n terms is S_n , show that

$$S_n = 2(T_n - 1). \quad [4]$$

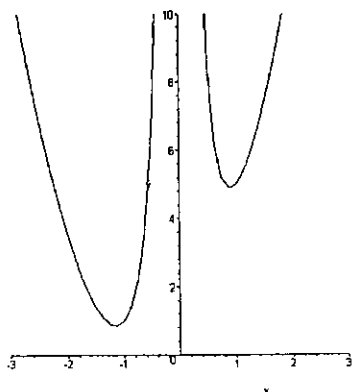
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8. The diagram shows a prism whose cross-section is a sector of a circle of radius r cm. The angle of the sector is θ radians. The length of the prism is $3r$ cm.



- (i) Show that the total surface area of the prism is $2r^2(3 + 2\theta)$ cm². [4]
 - (ii) Find an expression in terms of r and θ for the volume of the prism. [3]
- If the volume of the prism has a fixed value of 36 cm³,
- (iii) express the surface area in terms of r only. [3]
 - (iv) Show that the surface area is minimum when $r = 2$. [5]

9. The diagram shows the curve C with equation $y = f(x)$, where $x \neq 0$.



Given that $f'(x) = 4x + 2 - \frac{4}{x^3}$, and that C passes through the point $A(1, 5)$,

- (i) find $f(x)$. [4]
- (ii) Verify that C also passes through $B(-1, 1)$. [2]
- (iii) Show that the tangents to C at A and B are parallel. [3]
- (iv) Find the area between the curve C , the lines $x = -3$ and $x = -1$, and the x -axis. [6]

CORE MATHS 2 (C) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1. $BC^2 = 8 + 12.25 - 14 = 6.25$ $BC = 2.5$ cm M1 A1 M1 A1 4
2. $s = r + 1$, so $r = s - 1$ $as + b = 4(s - 1) + 2$ $a = 4, b = -2$ M1 A1 M1 A1 A1 5
3. (i) $\log_a(x^2/4) = 3$ $x^2 = 4a^3$ $x = 2a^{3/2}$ M1 A1 A1
- (ii) When $a = 9, x = 2 \times 27 = 54$ M1 A1 5
4. $(3 - 4x)^5 = 3^5 + 5(3^4)(-4x) + 10(3^3)(-4x)^2 + 10(3^2)(-4x)^3 + \dots$ M1 A1 A1
- $= 243 - 1620x + 4320x^2 - 5760x^3 + \dots$ M1 A1 A1 6
5. (i) (2, 0.549), (3, 0.691) B1 B1
- (ii) $\frac{1}{2}(\frac{1}{2})(0.738 + 2(2.291)) = 1.33$ M1 M1 A1 A1 6
6. (i) $x - 20 = 80, 100$ $x = 100, 120$ M1 A1 A1
- (ii) $3x = 120, 300$ $x = 40, 100, 160$ M1 A1 A1 A1 7
7. (i) $(p^2)^2 = p(p^2 + 4)$ $p^4 - p^3 - 4p = 0$ $p(p - 2)(p^2 + p + 2) = 0$ B1 M1 A1
- $p \neq 0$ (0, 0, 4 not a G.P.) and $p^2 + p + 2 = 0$ has no real roots, so $p = 2$ M1 A1
- (ii) $T_n = 2(2^{n-1}) = 2^n$ $S_n = 2(2^n - 1)/(2 - 1) = 2(T_n - 1)$ B1 M1 A1 A1 9
8. (i) S.A. = $(r + r + r\theta)(3r) + 2(\frac{1}{2}r^2\theta) = 6r^2 + 4r^2\theta = 2r^2(3 + 2\theta)$ cm² M1 A1 M1 A1
- (ii) $V = 3r(\frac{1}{2}r^2\theta) = 3/2 r^3\theta$ cm³ M1 A1 A1
- (iii) $3/2 r^3\theta = 36$ $\theta = 24/r^3$ S.A. = $6r^2 + 96/r$ M1 A1 A1
- (iv) d/dr (S.A.) = $12r - 96/r^2 = 0$ when $r^3 = 8$ $r = 2$ M1 A1 A1
- Second derivative = $12 + 192/r^3 > 0$, so minimum B2 15
9. (i) Integrating, $f(x) = 2x^2 + 2x + 2/x^2 + c$ $5 = 6 + c$, so $c = -1$ M1 A1 M1 A1
- (ii) When $x = -1, 2x^2 + 2x + 2/x^2 - 1 = 2 - 2 + 2 - 1 = 1$ M1 A1
- (iii) At A, gradient = $4 + 2 - 4 = 2$ At B, gradient = $-4 + 2 + 4 = 2$ M1 A1 A1
- (iv) Area = $\int_{-3}^{-1} f(x) dx = \left[\frac{2x^3}{3} + x^2 - \frac{2}{x} - x \right]_{-3}^{-1} = \frac{10}{3} - \left[-\frac{16}{3} \right] = \frac{26}{3}$ M1 A2 M1 A1 A1