General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MPC2

Unit Pure Core 2

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

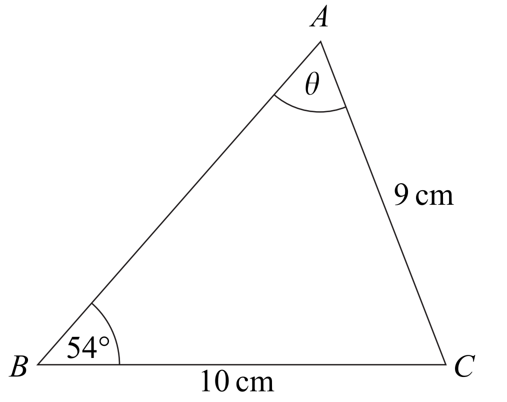
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The triangle ABC, shown in the diagram, is such that $AC = 9\text{ cm}$, $BC = 10\text{ cm}$, angle $ABC = 54°$ and the acute angle $BAC = \theta$.



(a) Show that $\theta = 64°$, correct to the nearest degree. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest square centimetre. *(3 marks)*

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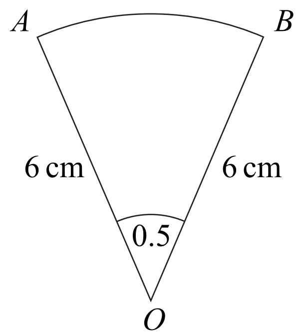
2 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle $AOB = 0.5$ radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the length of the arc AB. *(2 marks)*

(ii) Hence show that

the perimeter of the sector $OAB = k \times$ the length of the arc AB

where k is an integer. *(2 marks)*

3 (a) The expression $(2+x^2)^3$ can be written in the form

$$8+px^2+qx^4+x^6$$

Show that $p=12$ and find the value of the integer q. *(3 marks)*

(b) (i) Hence find $\int \frac{(2+x^2)^3}{x^4}\,dx$. *(5 marks)*

(ii) Hence find the exact value of $\int_1^2 \frac{(2+x^2)^3}{x^4}\,dx$. *(2 marks)*

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4 (a) Sketch the curve with equation $y=4^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

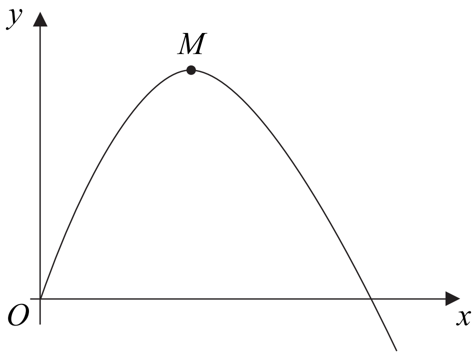
(b) Describe the geometrical transformation that maps the graph of $y=4^x$ onto the graph of $y=4^x-5$. *(2 marks)*

(c) (i) Use the substitution $Y=2^x$ to show that the equation $4^x-2^{x+2}-5=0$ can be written as $Y^2-4Y-5=0$. *(2 marks)*

(ii) Hence show that the equation $4^x-2^{x+2}-5=0$ has only one real solution.
Use logarithms to find this solution, giving your answer to three decimal places. *(4 marks)*

Turn over ▶

5 The diagram shows part of a curve with a maximum point M.



The curve is defined for $x \geqslant 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) (i) Hence find the coordinates of the maximum point M. *(3 marks)*

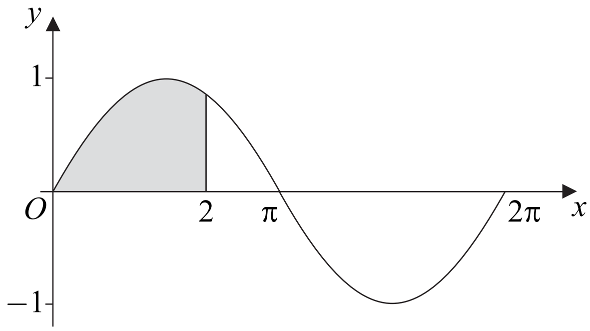
(ii) Write down the equation of the normal to the curve at M. *(1 mark)*

(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point P, giving your answer in the form $ax + by = c$, where a, b and c are positive integers. *(4 marks)*

(ii) The normals to the curve at the points M and P intersect at the point R. Find the coordinates of R. *(2 marks)*

6 A curve C, defined for $0 \leqslant x \leqslant 2\pi$ by the equation $y = \sin x$, where x is in radians, is sketched below. The region bounded by the curve C, the x-axis from 0 to 2 and the line $x = 2$ is shaded.



(a) The area of the shaded region is given by $\int_0^2 \sin x \, dx$, where x is in radians.

Use the trapezium rule with five ordinates (four strips) to find an approximate value for the area of the shaded region, giving your answer to three significant figures. *(4 marks)*

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = 2\sin x$. *(2 marks)*

(c) Use a trigonometrical identity to solve the equation

$$2\sin x = \cos x$$

in the interval $0 \leqslant x \leqslant 2\pi$, giving your solutions in radians to three significant figures. *(4 marks)*

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7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 60$ and $u_2 = 48$.

The limit of u_n as n tends to infinity is 12.

(a) Show that $p = \frac{3}{4}$ and find the value of q. *(5 marks)*

(b) Find the value of u_3. *(1 mark)*

Turn over ▶

8 Prove that, for all values of x, the value of the expression

$$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$$

is an integer and state its value. *(4 marks)*

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9 The first term of a geometric series is 12 and the common ratio of the series is $\frac{3}{8}$.

(a) Find the sum to infinity of the series. *(2 marks)*

(b) Show that the sixth term of the series can be written in the form $\dfrac{3^6}{2^{13}}$. *(3 marks)*

(c) The nth term of the series is u_n.

(i) Write down an expression for u_n in terms of n. *(1 mark)*

(ii) Hence show that

$$\log_a u_n = n\log_a 3 - (3n - 5)\log_a 2$$ *(4 marks)*

END OF QUESTIONS