C1 INTEGRATION — *Answers - Worksheet A*

1

a $\frac{1}{3}x^3 + c$ **b** $\frac{1}{7}x^7 + c$ **c** $\frac{1}{2}x^2 + c$ **d** $-\frac{1}{3}x^{-3} + c$ **e** $5x + c$ **f** $x^3 + c$

g $\frac{1}{2}x^8 + c$ **h** $-6x^{-1} + c$ **i** $\frac{4}{3}x^6 + c$ **j** $\frac{1}{6}x^2 + c$ **k** $-\frac{1}{4}x^{-8} + c$ **l** $-\frac{3}{8}x^{-2} + c$

2

a $= x^2 + 3x + c$

b $= 3x^4 - 2x^2 + c$

c $= 7x - \frac{1}{3}x^3 + c$

d $= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$

e $= \frac{1}{5}x^5 + \frac{5}{3}x^3 + c$

f $= \int (x^3 - 3x)\,dx$
$= \frac{1}{4}x^4 - \frac{3}{2}x^2 + c$

g $= \int (x^2 - 4x + 4)\,dx$
$= \frac{1}{3}x^3 - 2x^2 + 4x + c$

h $= \frac{3}{5}x^5 + \frac{1}{3}x^3 - 6x + c$

i $= \int (2 + x^{-2})\,dx$
$= 2x - x^{-1} + c$

j $= \int (x - x^{-3})\,dx$
$= \frac{1}{2}x^2 + \frac{1}{2}x^{-2} + c$

k $= \int (2x^{-2} - 3x^2)\,dx$
$= -2x^{-1} - x^3 + c$

l $= \int (x^2 - 8 + 16x^{-2})\,dx$
$= \frac{1}{3}x^3 - 8x - 16x^{-1} + c$

3

a $= \frac{2}{3}y^{\frac{3}{2}} + c$

b $= \frac{2}{7}y^{\frac{7}{2}} + c$

c $= 2y^{\frac{1}{2}} + c$

d $= 3y^{\frac{4}{3}} + c$

e $= \frac{4}{7}y^{\frac{7}{4}} + c$

f $= 15y^{\frac{1}{3}} + c$

g $= \int y^{\frac{1}{4}}\,dx$
$= \frac{4}{5}y^{\frac{5}{4}} + c$

h $= \int 7y^{-\frac{1}{2}}\,dx$
$= 14y^{\frac{1}{2}} + c$

i $= \int \frac{1}{2}y^{-2}\,dx$
$= -\frac{1}{2}y^{-1} + c$

j $= \int y^{\frac{3}{2}}\,dx$
$= \frac{2}{5}y^{\frac{5}{2}} + c$

k $= \int \frac{5}{2}y^{-4}\,dx$
$= -\frac{5}{6}y^{-3} + c$

l $= \int \frac{1}{3}y^{-\frac{1}{2}}\,dx$
$= \frac{2}{3}y^{\frac{1}{2}} + c$

4

a $= 2t^{\frac{3}{2}} - t + c$

b $= \int (2r + r^{\frac{1}{2}})\,dr$
$= r^2 + \frac{2}{3}r^{\frac{3}{2}} + c$

c $= \int (9p^2 - 6p + 1)\,dp$
$= 3p^3 - 3p^2 + p + c$

d $= 2x^2 + \frac{3}{4}x^{\frac{4}{3}} + c$

e $= \int (y^{-3} + y)\,dy$
$= -\frac{1}{2}y^{-2} + \frac{1}{2}y^2 + c$

f $= \frac{1}{6}x^3 - \frac{2}{5}x^{\frac{5}{2}} + c$

g $= \int (t^2 + 2)\,dt$
$= \frac{1}{3}t^3 + 2t + c$

h $= \frac{3}{8}r^{\frac{8}{3}} - \frac{3}{5}r^{\frac{5}{3}} + c$

i $= \int (2p^3 - \frac{1}{2}p)\,dp$
$= \frac{1}{2}p^4 - \frac{1}{4}p^2 + c$

j $= 4y - \frac{4}{11}y^{\frac{11}{4}} + c$

k $= \int (\frac{1}{3}x^{-2} + 2)\,dx$
$= -\frac{1}{3}x^{-1} + 2x + c$

l $= \int (2t^{\frac{1}{2}} + 3t^{-\frac{1}{2}})\,dt$
$= \frac{4}{3}t^{\frac{3}{2}} + 6t^{\frac{1}{2}} + c$

5 **a** $= x^3 - \frac{1}{2}x^2 + 6x + c$

b $= \frac{1}{7}x^7 - \frac{1}{4}x^4 + x^2 - 5x + c$

c $= \int (x^3 - x^2 - 2x)\ dx$
$= \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + c$

d $= \int (x + 4x^{\frac{1}{2}} + 4)\ dx$
$= \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$

e $= \int (2x^3 + 3x^2 - 8x - 12)\ dx$
$= \frac{1}{2}x^4 + x^3 - 4x^2 - 12x + c$

f $= \int (x^3 - 2x^{\frac{4}{3}} + 7x^{-2})\ dx$
$= \frac{1}{4}x^4 - \frac{6}{7}x^{\frac{7}{3}} - 7x^{-1} + c$

g $= \int (\frac{1}{4}x^{-3} - \frac{2}{3}x^{-2})\ dx$
$= -\frac{1}{8}x^{-2} + \frac{2}{3}x^{-1} + c$

h $= \int (1 - 4x^{-2} + 4x^{-4})\ dx$
$= x + 4x^{-1} - \frac{4}{3}x^{-3} + c$

i $= \int (x^4 + x^{\frac{5}{2}} - x^{\frac{3}{2}} - 1)\ dx$
$= \frac{1}{5}x^5 + \frac{2}{7}x^{\frac{7}{2}} - \frac{2}{5}x^{\frac{5}{2}} - x + c$

6 **a** $y = \int (8x + 3)\ dx$
$y = 4x^2 + 3x + c$

b $y = \int (\frac{1}{2}x^3 - x^2)\ dx$
$y = \frac{1}{8}x^4 - \frac{1}{3}x^3 + c$

c $y = \int \frac{4}{3}x^{-3}\ dx$
$y = -\frac{2}{3}x^{-2} + c$

d $y = \int (x^3 + 3x^2 + 3x + 1)\ dx$
$y = \frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + x + c$

e $y = \int (2x - 3x^{-\frac{1}{2}})\ dx$
$y = x^2 - 6x^{\frac{1}{2}} + c$

f $y = \int (x^{\frac{3}{2}} - 2x^{-\frac{3}{2}})\ dx$
$y = \frac{2}{5}x^{\frac{5}{2}} + 4x^{-\frac{1}{2}} + c$

g $y = \int (\frac{3}{2}x^{-2} - \frac{1}{2})\ dx$
$y = -\frac{3}{2}x^{-1} - \frac{1}{2}x + c$

h $y = \int (10x^{-3} - 2x^{-2})\ dx$
$y = -5x^{-2} + 2x^{-1} + c$

i $y = \int (3x^{\frac{1}{2}} - \frac{2}{3}x^{-\frac{1}{2}})\ dx$
$y = 2x^{\frac{3}{2}} - \frac{4}{3}x^{\frac{1}{2}} + c$

C1 INTEGRATION — Answers - Worksheet B

1 **a** $x^2 + x + c$

b $y = x^2 + x + c$

$(1, 5) \Rightarrow 5 = 1 + 1 + c$

$\therefore c = 3$

$y = x^2 + x + 3$

2 **a** $y = \int (3 - 6x)\ dx$

$y = 3x - 3x^2 + c$

$(2, 1) \Rightarrow 1 = 6 - 12 + c$

$\therefore c = 7$

$y = 3x - 3x^2 + 7$

b $y = \int (3x^2 - x)\ dx$

$y = x^3 - \frac{1}{2}x^2 + c$

$(4, 41) \Rightarrow 41 = 64 - 8 + c$

$\therefore c = -15$

$y = x^3 - \frac{1}{2}x^2 - 15$

c $y = \int (x^2 + 4x + 1)\ dx$

$y = \frac{1}{3}x^3 + 2x^2 + x + c$

$(-3, 4) \Rightarrow 4 = -9 + 18 - 3 + c$

$\therefore c = -2$

$y = \frac{1}{3}x^3 + 2x^2 + x - 2$

d $y = \int (7 - 5x - x^3)\ dx$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4 + c$

$(2, 0) \Rightarrow 0 = 14 - 10 - 4 + c$

$\therefore c = 0$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4$

e $y = \int (8x - 2x^{-2})\ dx$

$y = 4x^2 + 2x^{-1} + c$

$(\frac{1}{2}, -1) \Rightarrow -1 = 1 + 4 + c$

$\therefore c = -6$

$y = 4x^2 + 2x^{-1} - 6$

f $y = \int (3 - x^{\frac{1}{2}})\ dx$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + c$

$(4, 8) \Rightarrow 8 = 12 - \frac{16}{3} + c$

$\therefore c = \frac{4}{3}$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}$

3 $f(x) = \int (3 + 2x - x^2)\ dx$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 + c$

$(3, 5) \Rightarrow 5 = 9 + 9 - 9 + c$

$\therefore c = -4$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 - 4$

4 $y = \int (10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}})\ dx$

$y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$

$y = 0$ when $x = 7$

$\therefore 7 = 0 + 0 + c$

$c = 7$

$\therefore y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + 7$

when $x = 4$

$y = 4(32) - 4(2) + 7$

$y = 127$

5 **a** $f(x) = \int (2x^3 - x - 8)\ dx$

$f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x + c$

$(-1, 4) \Rightarrow 4 = \frac{1}{2} - \frac{1}{2} + 8 + c$

$\therefore c = -4$

$f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x - 4$

b at $x = 2$, $y = 8 - 2 - 16 - 4 = -14$

grad $= 16 - 2 - 8 = 6$

$\therefore y + 14 = 6(x - 2)$

$[y = 6x - 26]$

6 $f(x) = \int (3x^2 - 8x - 5)\ dx$

$f(x) = x^3 - 4x^2 - 5x + c$

$(0, 0) \Rightarrow 0 = 0 + c$

$\therefore c = 0$

$f(x) = x^3 - 4x^2 - 5x$

$= x(x^2 - 4x - 5)$

$= x(x + 1)(x - 5)$

crosses x-axis when $f(x) = 0$

$\therefore (-1, 0)$ and $(5, 0)$

7 **a** $y = \int (3x + 2x^{-2})\ dx$

$y = \frac{3}{2}x^2 - 2x^{-1} + c$

b $y = 8$ when $x = 2$

$\therefore 8 = 6 - 1 + c$

$c = 3$

$\therefore y = \frac{3}{2}x^2 - 2x^{-1} + 3$

when $x = \frac{1}{2}$

$y = \frac{3}{8} - 4 + 3$

$y = -\frac{5}{8}$

8 **a** $y = \int (3x^2 + kx)\ dx$

$y = x^3 + \frac{1}{2}kx^2 + c$

$(1, 6) \Rightarrow 6 = 1 + \frac{1}{2}k + c$

$5 = \frac{1}{2}k + c$ (1)

$(2, 1) \Rightarrow 1 = 8 + 2k + c$

$-7 = 2k + c$ (2)

(2) – (1) $-12 = \frac{3}{2}k$

$k = -8$

b sub. $-7 = -16 + c$

$c = 9$

$\therefore y = x^3 - 4x^2 + 9$

C1 INTEGRATION — Answers - Worksheet C

1 $= \frac{1}{3}x^3 + 4x^{\frac{3}{2}} - 3x + c$

2 **a** $f(x) = \int (1 - 6x^{-3})\ dx$

$= x + 3x^{-2} + c$

$(1, -2) \Rightarrow -2 = 1 + 3 + c$

$c = -6$

$\therefore\ f(x) = x - 6 + \frac{3}{x^2}$

b $x = 2 \Rightarrow y = -\frac{13}{4}$, grad $= \frac{1}{4}$

$\therefore$ grad of normal $= -4$

$\therefore\ y + \frac{13}{4} = -4(x - 2)$

$4y + 13 = -16x + 32$

$16x + 4y - 19 = 0$

3 **a** $f(x) = \int (3x^2 + 2x - 5)\ dx$

$= x^3 + x^2 - 5x + c$

$(3, 22) \Rightarrow 22 = 27 + 9 - 15 + c$

$c = 1$

$\therefore\ f(x) = x^3 + x^2 - 5x + 1$

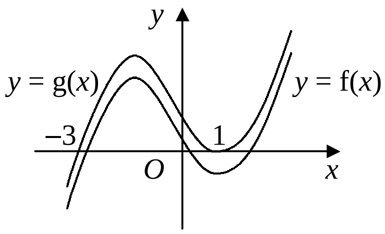
b $g(x) = (x + 3)(x^2 - 2x + 1)$

$= x^3 - 2x^2 + x + 3x^2 - 6x + 3$

$= x^3 + x^2 - 5x + 3$

$= f(x) + 2$

c



4 **a** $= 2x + 6x^{-3}$

b $= \frac{1}{3}x^3 + 3x^{-1} + c$

5 grad of tangent $= 12 - 8 - 1 = 3$

tangent passes through $(0, 0)$

$\therefore$ tangent: $y = 3x$

when $x = 2$, $y = 6$

$\therefore$ curve passes through $(2, 6)$

curve: $y = \int (3x^2 - 4x - 1)\ dx$

$y = x^3 - 2x^2 - x + c$

$(2, 6) \Rightarrow 6 = 8 - 8 - 2 + c$

$c = 8$

$\therefore\ y = x^3 - 2x^2 - x + 8$

6 **a** $= 3\sqrt{2} - \frac{2}{\sqrt{2}}$

$= 3\sqrt{2} - \sqrt{2}$

$= 2\sqrt{2}$

b $y = \int (3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}})\ dx$

$= 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$

$(4, 7) \Rightarrow 7 = 2(8) - 4(2) + c$

$7 = 16 - 8 + c$

$c = -1$

$\therefore\ y = 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - 1$

when $x = 3$

$y = 6\sqrt{3} - 4\sqrt{3} - 1$

$y = 2\sqrt{3} - 1$

7 **a** $= \int (x^2 + 4x + 4)\ dx$

$= \frac{1}{3}x^3 + 2x^2 + 4x + c$

b $= \int \frac{1}{4}x^{-\frac{1}{2}}\ dx$

$= \frac{1}{2}x^{\frac{1}{2}} + c$

8 **a** $f(x) = \int (3x^2 - 2x - 3)\ dx$

$= x^3 - x^2 - 3x + c$

$(-2, 0) \Rightarrow 0 = -8 - 4 + 6 + c$

$c = 6$

$\therefore f(x) = x^3 - x^2 - 3x + 6$

b $x = 1 \Rightarrow y = 1 - 1 - 3 + 6 = 3$

grad $= 3 - 2 - 3 = -2$

$\therefore y - 3 = -2(x - 1)$

$y - 3 = -2x + 2$

$y = 5 - 2x$

9 **a** $y = \int (2x - 3x^{-2})\ dx$

$= x^2 + 3x^{-1} + c$

$y = 0$ at $x = 1$

$\therefore 0 = 1 + 3 + c$

$c = -4$

$\therefore y = x^2 - 4 + \frac{3}{x}$

b $\frac{d^2y}{dx^2} = 2 + 6x^{-3}$

$\therefore x^2\frac{d^2y}{dx^2} - 2y$

$= x^2(2 + 6x^{-3}) - 2(x^2 - 4 + 3x^{-1})$

$= 2x^2 + 6x^{-1} - 2x^2 + 8 - 6x^{-1}$

$= 8$ [$k = 8$]

10 **a** $= -\frac{1}{2}x^{-2} + c$

b $= \int \frac{x^2 - 2x + 1}{x^{\frac{1}{2}}}\ dx$

$= \int (x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}})\ dx$

$= \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

11 **a** $f(x) = \int (4x^3 - 8x)\ dx$

$= x^4 - 4x^2 + c$

$(2, -5) \Rightarrow -5 = 16 - 16 + c$

$c = -5$

$\therefore f(x) = x^4 - 4x^2 - 5$

b $x^4 - 4x^2 - 5 = 0$

$(x^2 + 1)(x^2 - 5) = 0$

$x^2 = -1$ [no sols] or 5

$x = \pm\sqrt{5}$

$\therefore (-\sqrt{5}, 0), (\sqrt{5}, 0)$

12 **a** $y = \int (k - x^{-\frac{1}{2}})\ dx$

$y = kx - 2x^{\frac{1}{2}} + c$

$(1, -2) \Rightarrow -2 = k - 2 + c$

$0 = k + c$ (1)

$(4, 5) \Rightarrow 5 = 4k - 4 + c$

$9 = 4k + c$ (2)

(2) − (1) $9 = 3k$

$k = 3$

b grad $= 3 - 1 = 2$

$\therefore$ grad of normal $= -\frac{1}{2}$

$\therefore y + 2 = -\frac{1}{2}(x - 1)$

$2y + 4 = -x + 1$

$x + 2y + 3 = 0$