C1 GRAPHS OF FUNCTIONS — Worksheet A

1 Sketch and label each pair of graphs on the same set of axes showing the coordinates of any points where the graphs intersect. Write down the equations of any asymptotes.

a $y = x^2$ and $y = x^3$ **b** $y = x^2$ and $y = x^4$

c $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$ **d** $y = x$ and $y = \sqrt{x}$

e $y = x^2$ and $y = 3x^2$ **f** $y = \frac{1}{x}$ and $y = \frac{2}{x}$

2 $f(x) = (x - 1)(x - 3)(x - 4)$.

a Find f(0).

b Write down the solutions of the equation $f(x) = 0$.

c Sketch the curve $y = f(x)$.

3 Sketch each graph showing the coordinates of any points of intersection with the coordinate axes.

a $y = (x + 1)(x - 1)(x - 3)$ **b** $y = 2x(x - 1)(x - 5)$

c $y = -(x + 2)(x + 1)(x - 2)$ **d** $y = x^2(x - 4)$

e $y = 3x(2 + x)(1 - x)$ **f** $y = (x + 2)(x - 1)^2$

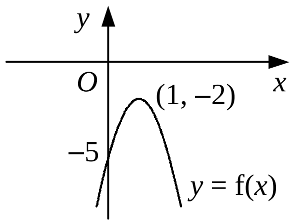
4 **a** Factorise fully $x^3 + 6x^2 + 9x$.

b Hence, sketch the curve $y = x^3 + 6x^2 + 9x$, showing the coordinates of any points where the curve meets the coordinate axes.

5 Given that the constants p and q are such that $p > q > 0$, sketch each of the following graphs showing the coordinates of any points of intersection with the coordinate axes.

a $y = (x - p)(x - q)^2$ **b** $y = (x - p)(x^2 - q^2)$

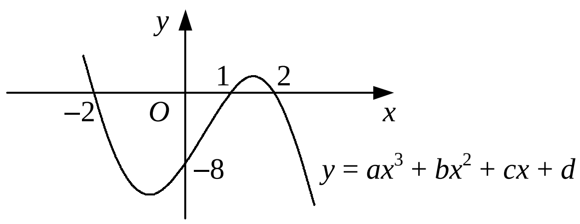
6



The diagram shows the curve with equation $y = f(x)$ which has a turning point at $(1, -2)$ and crosses the y-axis at the point $(0, -5)$.

Given that f(x) is a quadratic function, find an expression for f(x).

7



The diagram shows the curve with equation $y = ax^3 + bx^2 + cx + d$.

Given that the curve crosses the y-axis at the point $(0, -8)$ and crosses the x-axis at the points $(-2, 0)$, $(1, 0)$ and $(2, 0)$, find the values of the constants a, b, c and d.

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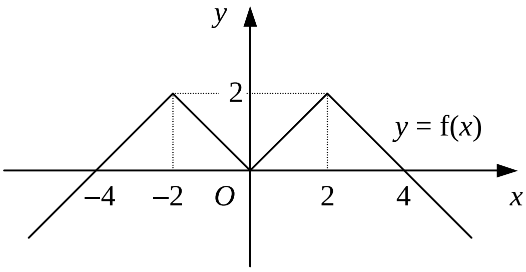


The diagram shows the graph of $y = \mathrm{f}(x)$.

Use the graph to write down the number of solutions that exist to each of the following equations.

a $\mathrm{f}(x) = 1$ **b** $\mathrm{f}(x) = 3$ **c** $\mathrm{f}(x) = -1$ **d** $\mathrm{f}(x) = 0$

9 **a** Sketch on the same set of axes the graphs of $y = x^2$ and $y = 1 - 2x$.

b Hence state the number of roots that the equation $x^2 + 2x - 1 = 0$ has and give a reason for your answer.

10 **a** Find the coordinates of the turning point of the curve $y = x^2 + 2x - 3$.

b By sketching two suitable graphs on the same set of axes, show that the equation

$$x^2 + 2x - 3 - \frac{1}{x} = 0$$

has one positive and two negative real roots.

11 Show that the line $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$.

12 **a** Solve the simultaneous equations

$$y = 3x + 7$$
$$y = x^2 + 5x + 8$$

b Hence, describe the geometrical relationship between the straight line $y = 3x + 7$ and the curve $y = x^2 + 5x + 8$.

13 **a** Find the coordinates of the points where the straight line $y = x + 6$ meets the curve $y = x^3 - 4x^2 + x + 6$.

b Given that

$$x^3 - 4x^2 + x + 6 \equiv (x + 1)(x - 2)(x - 3),$$

sketch the straight line $y = x + 6$ and the curve $y = x^3 - 4x^2 + x + 6$ on the same diagram, showing the coordinates of the points where the curve crosses the coordinate axes.

14 Find the value of the constant k such that the straight line with equation $y = 3x + k$ is a tangent to the curve with equation $y = 2x^2 - 5x + 1$.

15 Find the set of values of the constant a for which the line $y = 2 - 5x$ intersects the curve $y = x^2 + ax + 18$ at two points.

16 The curve C has the equation $y = x^2 - 2x + 6$.

a Find the values of p for which the line $y = px + p$ is a tangent to the curve C.

b Prove that there are no real values of q for which the line $y = qx + 7$ is a tangent to the curve C.

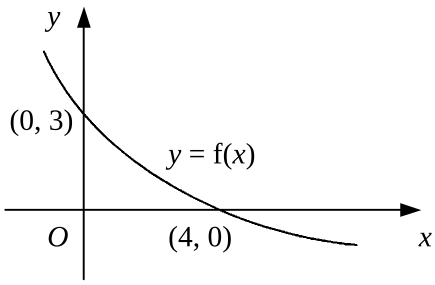
C1 GRAPHS OF FUNCTIONS Worksheet B

1 Describe how the graph of $y = \mathrm{f}(x)$ is transformed to give the graph of

a $y = \mathrm{f}(x - 1)$ **b** $y = \mathrm{f}(x) - 3$ **c** $y = 2\mathrm{f}(x)$ **d** $y = \mathrm{f}(4x)$

e $y = -\mathrm{f}(x)$ **f** $y = \frac{1}{5}\mathrm{f}(x)$ **g** $y = \mathrm{f}(-x)$ **h** $y = \mathrm{f}(\frac{2}{3}x)$

2



The diagram shows the curve with equation $y = \mathrm{f}(x)$ which crosses the coordinate axes at the points (0, 3) and (4, 0).

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = 3\mathrm{f}(x)$ **b** $y = \mathrm{f}(x + 4)$ **c** $y = -\mathrm{f}(x)$ **d** $y = \mathrm{f}(\frac{1}{2}x)$

3 Find and simplify an equation of the graph obtained when

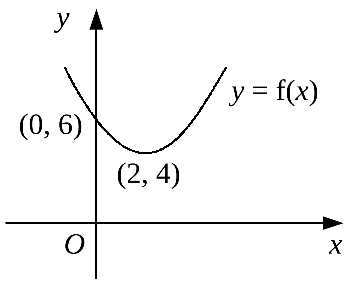
a the graph of $y = 2x + 5$ is translated by 1 unit in the positive y-direction,

b the graph of $y = 1 - 4x$ is stretched by a factor of 3 in the y-direction, about the x-axis,

c the graph of $y = 3x + 1$ is translated by 4 units in the negative x-direction,

d the graph of $y = 4x - 7$ is reflected in the x-axis.

4



The diagram shows the curve with equation $y = \mathrm{f}(x)$ which has a turning point at (2, 4) and crosses the y-axis at the point (0, 6).

Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = \mathrm{f}(x) - 3$ **b** $y = \mathrm{f}(x + 2)$ **c** $y = \mathrm{f}(2x)$ **d** $y = \frac{1}{2}\mathrm{f}(x)$

5 Describe a single transformation that would map the graph of $y = x^3$ onto the graph of

a $y = 4x^3$ **b** $y = (x - 2)^3$ **c** $y = -x^3$ **d** $y = x^3 + 5$

6 Describe a single transformation that would map the graph of $y = x^2 + 2$ onto the graph of

a $y = 2x^2 + 4$ **b** $y = x^2 - 5$ **c** $y = \frac{1}{9}x^2 + 2$ **d** $y = x^2 + 4x + 6$

7 Find and simplify an equation of the graph obtained when

a the graph of $y = x^2 + 2x$ is translated by 1 unit in the positive x-direction,

b the graph of $y = x^2 - 4x + 5$ is stretched by a factor of $\frac{1}{3}$ in the x-direction, about the y-axis.

c the graph of $y = x^2 + x - 6$ is reflected in the y-axis,

d the graph of $y = 2x^2 - 3x$ is stretched by a factor of 2 in the x-direction, about the y-axis.

8 $$f(x) \equiv x^2 - 4x.$$

a Find the coordinates of the turning point of the graph $y = f(x)$.

b Sketch each pair of graphs on the same set of axes showing the coordinates of the turning point of each graph.

i $y = f(x)$ and $y = 3 + f(x)$ **ii** $y = f(x)$ and $y = f(x - 2)$ **iii** $y = f(x)$ and $y = f(2x)$

9 Sketch each pair of graphs on the same set of axes.

a $y = x^2$ and $y = (x + 3)^2$ **b** $y = x^3$ and $y = x^3 + 4$

c $y = \frac{1}{x}$ and $y = \frac{1}{x-2}$ **d** $y = \sqrt{x}$ and $y = \sqrt{2x}$

10 a Describe two different transformations, each of which would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{1}{3x}$.

b Describe two different transformations, each of which would map the graph of $y = x^2$ onto the graph of $y = 4x^2$.

11 $$f(x) \equiv (x + 4)(x + 2)(x - 1).$$

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

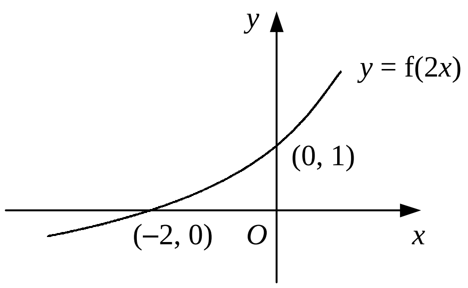
a $y = f(x)$ **b** $y = f(x - 4)$ **c** $y = f(-x)$ **d** $y = f(2x)$

12 The curve $y = f(x)$ is a parabola and the coordinates of its turning point are (a, b).

Write down, in terms of a and b, the coordinates of the turning point of the graph

a $y = 3f(x)$ **b** $y = 4 + f(x)$ **c** $y = f(x + 1)$ **d** $y = f(\frac{1}{3}x)$

13



The diagram shows the curve with equation $y = f(2x)$ which crosses the coordinate axes at the points $(-2, 0)$ and $(0, 1)$.

Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the curves

a $y = 3f(2x)$ **b** $y = f(x)$

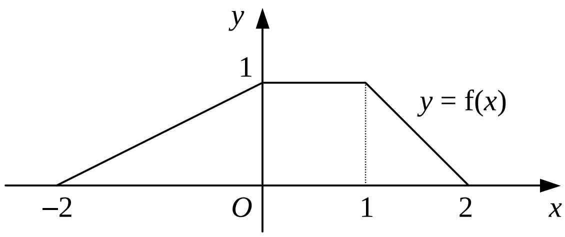
C1 GRAPHS OF FUNCTIONS Worksheet C

1 **a** Solve the simultaneous equations

$$y = 3x - 4$$
$$y = 4x^2 - 9x + 5$$ **(4)**

b Hence, describe the geometrical relationship between the straight line $y = 3x - 4$ and the curve $y = 4x^2 - 9x + 5$. **(1)**

2



The diagram shows the graph of $y = \mathrm{f}(x)$ which is defined for $-2 \le x \le 2$.

Labelling the axes in a similar way, sketch on separate diagrams the graphs of

a $y = 3\mathrm{f}(x)$, **(2)**

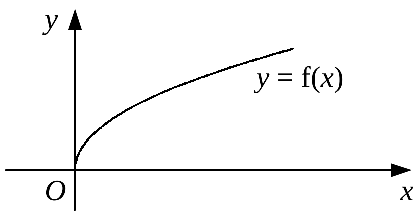
b $y = \mathrm{f}(x + 1)$, **(2)**

c $y = \mathrm{f}(-x)$. **(2)**

3 **a** Show that the line $y = 4x + 1$ does not intersect the curve $y = x^2 + 5x + 2$. **(4)**

b Find the values of m such that the line $y = mx + 1$ meets the curve $y = x^2 + 5x + 2$ at exactly one point. **(4)**

4



The diagram shows the curve with the equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) \equiv \sqrt{x}, \quad x \ge 0.$$

a Sketch on the same set of axes the graphs of $y = 1 + \mathrm{f}(x)$ and $y = \mathrm{f}(x + 3)$. **(4)**

b Find the coordinates of the point of intersection of the two graphs drawn in part **a**. **(4)**

5 The curve C has the equation $y = x^2 + kx - 3$ and the line l has the equation $y = k - x$, where k is a constant.

Prove that for all real values of k, the line l will intersect the curve C at exactly two points. **(7)**

6
$$\mathrm{f}(x) \equiv 2x^2 - 4x + 5.$$

a Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$. **(3)**

b Showing the coordinates of the turning point in each case, sketch on the same set of axes the curves

i $y = \mathrm{f}(x)$,

ii $y = \mathrm{f}(x + 3)$. **(4)**

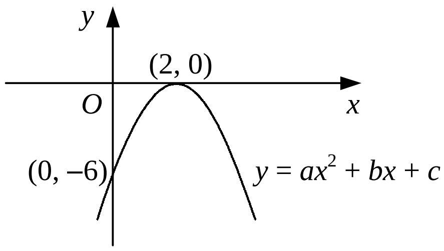
7 **a** Sketch on the same diagram the straight line $y = 2x - 5$ and the curve $y = x^3 - 3x^2$, showing the coordinates of any points where each graph meets the coordinate axes. **(4)**

b Hence, state the number of real roots that exist for the equation

$$x^3 - 3x^2 - 2x + 5 = 0,$$

giving a reason for your answer. **(2)**

8



The diagram shows the curve with the equation $y = ax^2 + bx + c$.

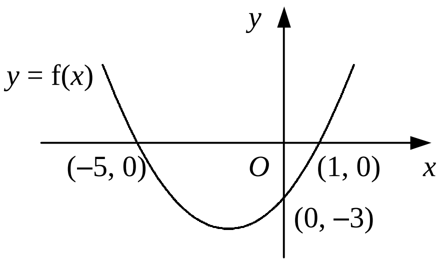
Given that the curve crosses the y-axis at the point (0, –6) and touches the x-axis at the point (2, 0), find the values of the constants a, b and c. **(6)**

9 **a** Show that

$$(1 - x)(2 + x)^2 \equiv 4 - 3x^2 - x^3.$$ **(3)**

b Hence, sketch the curve $y = 4 - 3x^2 - x^3$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

10



The diagram shows the curve with equation $y = \mathrm{f}(x)$ which crosses the coordinate axes at the points (–5, 0), (1, 0) and (0, –3).

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the curves

a $y = -\mathrm{f}(x)$, **(2)**

b $y = \mathrm{f}(x - 5)$, **(2)**

c $y = \mathrm{f}(2x)$. **(2)**

11 **a** Describe fully the transformation that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(x + 1)$. **(2)**

b Sketch the graph of $y = \dfrac{1}{x+1}$, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. **(3)**

c By sketching another suitable curve on your diagram in part **b**, show that the equation

$$x^3 - \frac{1}{x+1} = 2$$

has one positive and one negative real root. **(4)**