In this Booklet:

1. Hardest Past paper questions between 2009-2015
2. Hardest mixed exercise questions from the text book
3. Hardest Solomon paper questions

Answers can all be found individually on Physics and Maths tutor, including mixed exercises solutions in the solution bank section.

<u>But</u> complete answer booklet for this can be found on StudentGrounds.

Hardest Past Paper Questions

June 2015

7. Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

10. A curve with equation $y = f(x)$ passes through the point (4, 9).

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

(a) find $f(x)$, giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$

(b) Find the x coordinate of P.

(5)

January 2014

6.

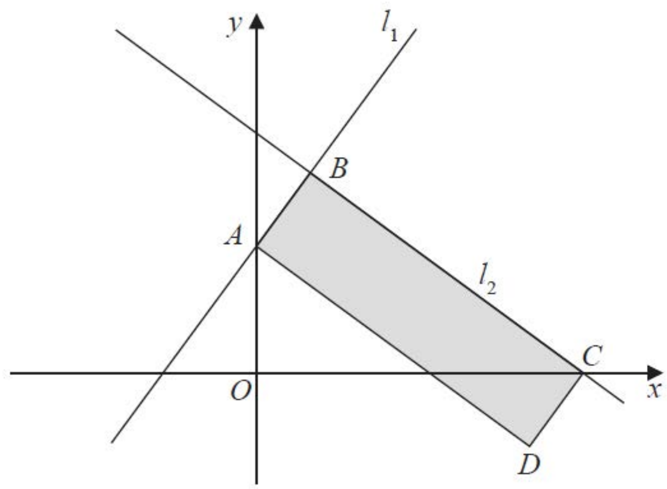

Figure 2

The straight line l_1 has equation $2y = 3x + 7$.

The line l_1 crosses the y-axis at the point A as shown in Figure 2.

(*a*) (i) State the gradient of l_1.

(ii) Write down the coordinates of the point A.

(2)

Another straight line l_2 intersects l_1 at the point B (1, 5) and crosses the x-axis at the point C, as shown in Figure 2.

Given that $<ABC = 90°$,

(*b*) find an equation of l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The rectangle $ABCD$, shown shaded in Figure 2, has vertices at the points A, B, C and D.

(*c*) Find the exact area of rectangle $ABCD$.

(5)

5. Given that for all positive integers n,

$$\sum_{r=1}^{n} a_r = 12 + 4n^2$$

(a) find the value of $\sum_{r=1}^{5} a_r$

(2)

(b) find the value of a_6

8. The equation $2x^2 + 2kx + (k + 2) = 0$, where k is a constant, has two distinct real roots.

(*a*) Show that k satisfies

$$k^2 - 2k - 4 > 0$$

(3)

(*b*) Find the set of possible values of k.

(4)

January 2013

2. Express 8^{2x+3} in the form 2^y, stating y in terms of x.

9. The equation

$$(k + 3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0$$

(4)

(b) Hence find the set of possible values of k.

(3)

10. $$4x^2 + 8x + 3 \equiv a(x + b)^2 + c$$

(a) Find the values of the constants a, b and c.

(3)

(b) On the axes on page 27, sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

June 2013 R

5. Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

(4)

6. A sequence $x_1, x_2, x_3 \dots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = (x_n)^2 - kx_n, \quad n \geqslant 1$$

where k is a constant, $k \neq 0$

(a) Find an expression for x_2 in terms of k.

(1)

(b) Show that $x_3 = 1 - 3k + 2k^2$

(2)

Given also that $x_3 = 1$,

(c) calculate the value of k.

(3)

(d) Hence find the value of $\sum_{n=1}^{100} x_n$

(3)

7. Each year, Abbie pays into a savings scheme. In the first year she pays in £500. Her payments then increase by £200 each year so that she pays £700 in the second year, £900 in the third year and so on.

(a) Find out how much Abbie pays into the savings scheme in the tenth year.

(2)

Abbie pays into the scheme for n years until she has paid in a total of £67 200.

(b) Show that $n^2 + 4n - 24 \times 28 = 0$

(5)

(c) Hence find the number of years that Abbie pays into the savings scheme.

(2)

8. A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m^2,

(b) (i) write down an inequality, in terms of x, for the area of the room.

(ii) Solve this inequality.

(4)

(c) Hence find the range of possible values for x.

(1)

9.

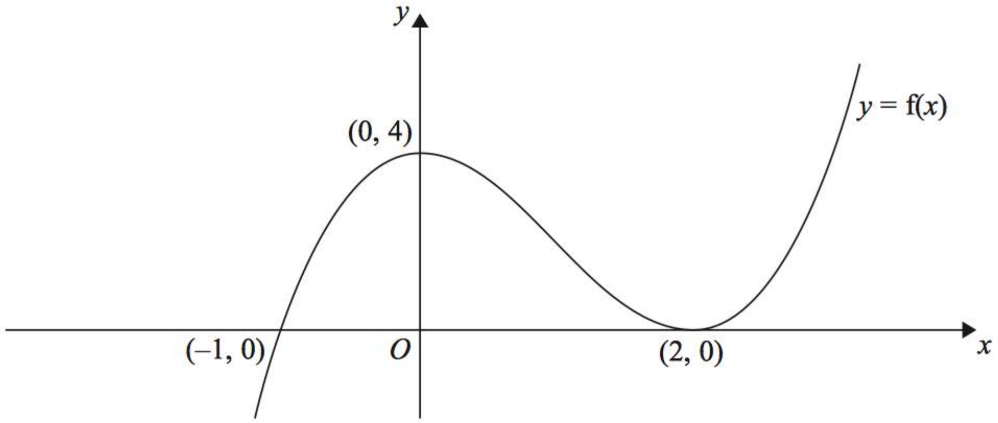

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

The curve C passes through the point (–1, 0) and touches the x-axis at the point (2, 0).

The curve C has a maximum at the point (0, 4).

(a) The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where a, b and c are integers.

Calculate the values of a, b and c.

(5)

(b) Sketch the curve with equation $y = \mathrm{f}(\frac{1}{2}x)$ in the space provided on page 24

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

(3)

10. A curve has equation $y = \mathrm{f}(x)$. The point P with coordinates (9, 0) lies on the curve.

Given that

$$\mathrm{f}'(x) = \frac{x + 9}{\sqrt{x}}, \qquad x > 0$$

(a) find $\mathrm{f}(x)$.

(6)

(b) Find the x-coordinates of the two points on $y = \mathrm{f}(x)$ where the gradient of the curve is equal to 10

(4)

11.

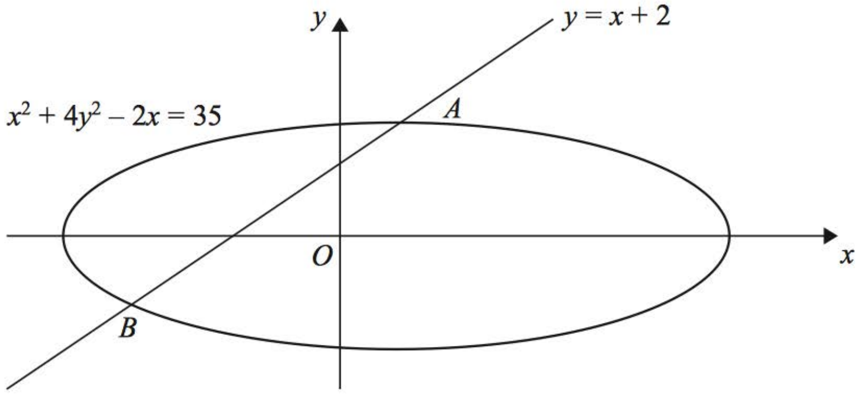

Figure 2

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

(a) Find the coordinates of A and the coordinates of B.

(6)

(b) Find the distance AB in the form $r\sqrt{2}$ where r is a rational number.

(3)

June 2013 paper

10. Given the simultaneous equations

$$2x + y = 1$$
$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

11.

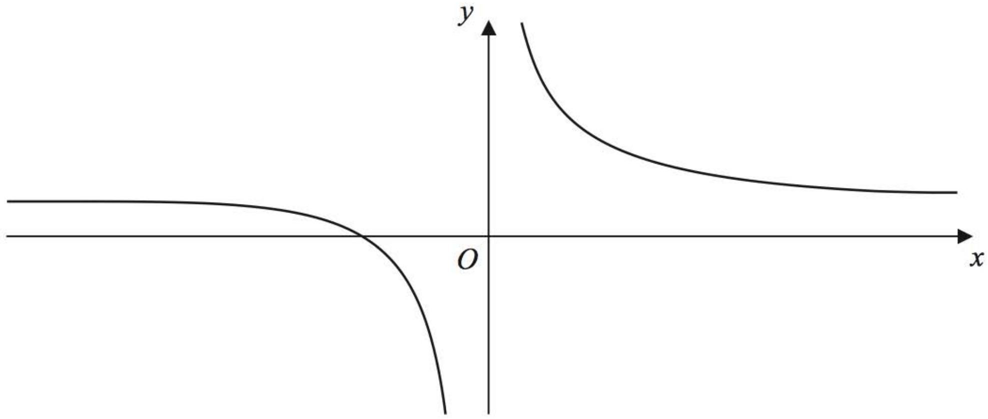

Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4, \ x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.
(1)

(b) Give the equations of the asymptotes to H.
(2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$.
(5)

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd.
(3)

StudentGrounds

January 2012

2 b)

(b) Simplify

$$\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$$

giving your answer in the form $b\sqrt{2}+c$, where b and c are integers.

(4)

3b) (b) $x(x-4)>12$

6.

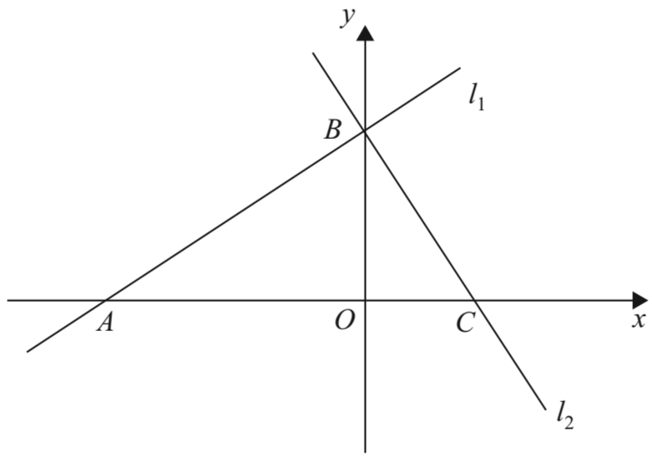

Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$

(a) Find the gradient of l_1.

(1)

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2.

(3)

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC.

(4)

8. The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

June 2012

8. $$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q.

(3)

(b) Calculate the discriminant of $4x - 5 - x^2$

(2)

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

9. The line L_1 has equation $4y + 3 = 2x$

The point A $(p, 4)$ lies on L_1

(a) Find the value of the constant p.

(1)

The line L_2 passes through the point C (2, 4) and is perpendicular to L_1

(b) Find an equation for L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The line L_1 and the line L_2 intersect at the point D.

(c) Find the coordinates of the point D.

(3)

(d) Show that the length of CD is $\frac{3}{2}\sqrt{5}$

(3)

A point B lies on L_1 and the length of $AB = \sqrt{(80)}$

The point E lies on L_2 such that the length of the line CDE = 3 times the length of CD.

(e) Find the area of the quadrilateral $ACBE$.

(3)

Jan 2011 :

1b) (*b*) Simplify $x\left(2x^{-\frac{1}{4}}\right)^{4}$.

(2)

5.

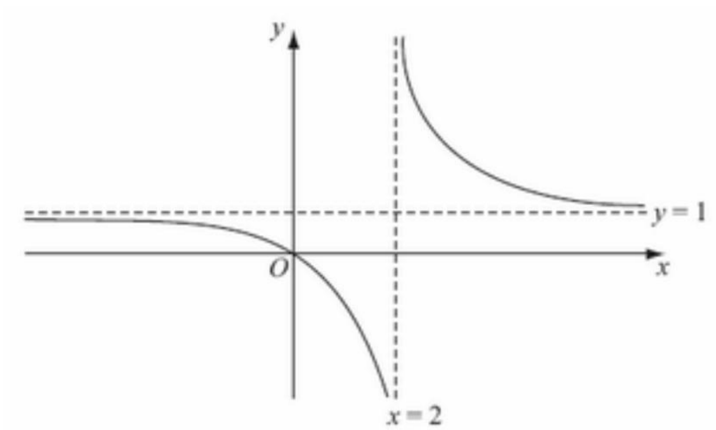

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2.$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(*a*) In the space below, sketch the curve with equation $y = \mathrm{f}(x - 1)$ and state the equations of the asymptotes of this curve.

(3)

(*b*) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x - 1)$ crosses the coordinate axes.

(4)

11. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(*b*) Show that the point $P(4, -8)$ lies on C.

(2)

(*c*) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

June 2011

3. The points P and Q have coordinates (–1, 6) and (9, 0) respectively.
The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax+by+c=0$, where a, b and c are integers.

(5)

6. Given that $\frac{6x+3x^2}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

7. $$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k+a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

8.

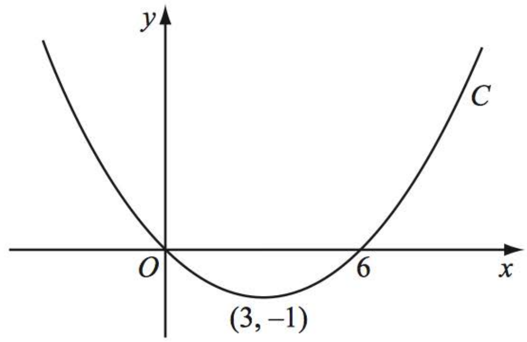

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$,

(3)

(b) $y = -f(x)$,

(3)

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$.

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

9. (a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \ldots\ldots + 100$$

(3)

(b) In the arithmetic series

$$k + 2k + 3k + \ldots\ldots + 100$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}$$

(4)

(c) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1),\ (4k + 4),\ (6k + 7),\ \ldots\ldots ,$$

giving your answer in its simplest form.

(2)

10. The curve C has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B.

(3)

January 2010

4.

$$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0.$$

Given that $y = 35$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

10

$$f(x) = x^2 + 4kx + (3 + 11k), \quad \text{where } k \text{ is a constant.}$$

(*a*) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation f(x) = 0 has no real roots,

(*b*) find the set of possible values of k.

(4)

Given that $k = 1$,

(*c*) sketch the graph of y = f(x), showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

June 2010

3) Find the set of values of x for which

(c) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$.

10. (a) On the axes below sketch the graphs of

(i) $y = x(4 - x)$

(ii) $y = x^2(7 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(b) Show that the x-coordinates of the points of intersection of

$$y = x(4 - x) \quad \text{and} \quad y = x^2(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3},\ r + s\sqrt{3})$, where p, q, r and s are integers.

(7)

January 2009

9. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.

(2)

(b) Show that $a = -17.5$ and find the value of d.

(2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(d) Hence find the value of n.

(3)

11. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0\,.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB.

(4)

June 2009

2. Given that $32\sqrt{2} = 2^a$, find the value of a.

(3)

5. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d,

(3)

(b) the value of a,

(2)

(c) the total number of houses built in Oldtown over the 40-year period.

(3)

6. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)

Hardest Mixed Exercises Questions

Equations and inequalities
Exercise F, Question 13

Question:

The specification for a rectangular car park states that the length x m is to be 5 m more than the breadth. The perimeter of the car park is to be greater than 32 m.

(a) Form a linear inequality in x.
The area of the car park is to be less than 104m^2.

(b) Form a quadratic inequality in x.

(c) By solving your inequalities, determine the set of possible values of x. **[E]**

Sketching curves
Exercise H, Question 2

Question:

(a) On the same axes sketch the curves with equations $y = \frac{6}{x}$ and $y = 1 + x$.

(b) The curves intersect at the points A and B. Find the coordinates of A and B.

(c) The curve C with equation $y = x^2 + px + q$, where p and q are integers, passes through A and B. Find the values of p and q.

(d) Add C to your sketch.

Sequences and series
Exercise H, Question 11

Question:

A polygon has 10 sides. The lengths of the sides, starting with the smallest, form an arithmetic series. The perimeter of the polygon is 675 cm and the length of the longest side is twice that of the shortest side. Find, for this series:

(a) The common difference.

(b) The first term. **[E]**

Differentiation

Exercise I, Question 14

Question:

The curve with equation $y = ax^2 + bx + c$ passes through the point (1 , 2) . The gradient of the curve is zero at the point (2 , 1) . Find the values of a, b and c. **[E]**

Hardest Solomon Paper Questions

Solomon paper A

4. *(a)* Solve the inequality

$$x^2 + 3x > 10.$$ **(3)**

(b) Find the set of values of x which satisfy both of the following inequalities:

$$3x - 2 < x + 3$$

$$x^2 + 3x > 10$$ **(3)**

Solomon paper B

6. *(a)* By completing the square, find in terms of the constant k the roots of the equation

$$x^2 + 4kx - k = 0.$$ **(4)**

(b) Hence find the set of values of k for which the equation has no real roots. **(4)**

5. $$f(x) = 4x - 3x^2 - x^3.$$

(a) Fully factorise $4x - 3x^2 - x^3$. **(3)**

(b) Sketch the curve $y = f(x)$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

7. (*a*) Given that $y = 2^x$, find expressions in terms of y for

(*i*) 2^{x+2},

(*ii*) 2^{3-x}. **(4)**

(*b*) Show that using the substitution $y = 2^x$, the equation

$$2^{x+2} + 2^{3-x} = 33$$

can be rewritten as

$$4y^2 - 33y + 8 = 0.$$ **(2)**

(*c*) Hence solve the equation

$$2^{x+2} + 2^{3-x} = 33.$$ **(4)**

Solomon C

10. The curve C has the equation $y = f(x)$ where

$$f(x) = (x + 2)^3.$$

(a) Sketch the curve C, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

7. The first three terms of an arithmetic series are $(12 - p)$, $2p$ and $(4p - 5)$ respectively, where p is a constant.

(a) Find the value of p. **(2)**

Solomon E

4. A sequence of terms is defined by

$$u_n = 3^n - 2, \quad n \geq 1.$$

(a) Write down the first four terms of the sequence. **(2)**

The same sequence can also be defined by the recurrence relation

$$u_{n+1} = au_n + b, \quad n \geq 1, \quad u_1 = 1,$$

where a and b are constants.

(b) Find the values of a and b. **(4)**

6. $$f(x) = 2x^2 - 4x + 1.$$

(a) Find the values of the constants a, b and c such that

$$f(x) = a(x + b)^2 + c.$$ **(4)**

(b) State the equation of the line of symmetry of the curve $y = f(x)$. **(1)**

(c) Solve the equation $f(x) = 3$, giving your answers in exact form. **(3)**

9. A store begins to stock a new range of DVD players and achieves sales of £1500 of these products during the first month.

In a model it is assumed that sales will decrease by £x in each subsequent month, so that sales of £$(1500 - x)$ and £$(1500 - 2x)$ will be achieved in the second and third months respectively.

Given that sales total £8100 during the first six months, use the model to

(a) find the value of x, **(4)**

(b) find the expected value of sales in the eighth month, **(2)**

(c) show that the expected total of sales in pounds during the first n months is given by $kn(51 - n)$, where k is an integer to be found. **(3)**

(d) Explain why this model cannot be valid over a long period of time. **(1)**

StudentGrounds

Solomon F

1. Find in exact form the real solutions of the equation

$$x^4 = 5x^2 + 14.$$ **(3)**

9. The third term of an arithmetic series is $5\frac{1}{2}$.

The sum of the first four terms of the series is $22\frac{3}{4}$.

(a) Show that the first term of the series is $6\frac{1}{4}$ and find the common difference. **(7)**

(b) Find the number of positive terms in the series. **(3)**

(c) Hence, find the greatest value of the sum of the first *n* terms of the series. **(2)**

10. The curve C has the equation $y = \mathrm{f}(x)$.

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - \frac{2}{x^3}, \quad x \neq 0,$$

and that the point $P\ (1, 1)$ lies on C,

(a) find an equation for the tangent to C at P in the form $y = mx + c$, **(3)**

(b) find an equation for C, **(5)**

(c) find the x-coordinates of the points where C meets the x-axis, giving your answers in the form $k\sqrt{2}$. **(5)**

Solomon G

1. Solve the equation

$$9^x = 3^{x+2}.$$ **(3)**

3. The curve C has the equation $y = (x - a)^2$ where a is a constant.

Given that

$$\frac{dy}{dx} = 2x - 6,$$

(a) find the value of a, **(4)**

(b) describe fully a single transformation that would map C onto the graph of $y = x^2$. **(2)**

6. $$f(x) = 9 + 6x - x^2.$$

(a) Find the values of A and B such that

$$f(x) = A - (x + B)^2.$$ **(4)**

(b) State the maximum value of $f(x)$. **(1)**

(c) Solve the equation $f(x) = 0$, giving your answers in the form $a + b\sqrt{2}$ where a and b are integers. **(3)**

(d) Sketch the curve $y = f(x)$. **(2)**

7. *(a)* An arithmetic series has a common difference of 7.

Given that the sum of the first 20 terms of the series is 530, find

(i) the first term of the series,

(ii) the smallest positive term of the series. **(5)**

(b) The terms of a sequence are given by

$$u_n = (n + k)^2, \quad n \geq 1,$$

where k is a positive constant.

Given that $u_2 = 2u_1$,

(i) find the value of k,

(ii) show that $u_3 = 11 + 6\sqrt{2}$. **(6)**

Solomon H

1. Evaluate

$$\sum_{r=1}^{30} (3r + 4).$$ **(3)**

8. *(a)* Prove that the sum of the first n positive integers is given by

$$\tfrac{1}{2}n(n + 1).$$ **(4)**

(b) Hence, find the sum of

(i) the integers from 100 to 200 inclusive,

(ii) the integers between 300 to 600 inclusive which are divisible by 3. **(5)**

9. *(a)* Express each of the following in the form $p + q\sqrt{2}$ where p and q are rational.

(i) $(4 - 3\sqrt{2})^2$

(ii) $\dfrac{1}{2+\sqrt{2}}$ **(5)**

(b) *(i)* Solve the equation

$$y^2 + 8 = 9y.$$

(ii) Hence solve the equation

$$x^3 + 8 = 9x^{\frac{3}{2}}.$$ **(5)**

Solomon I

1. The nth term of a sequence is defined by

$$u_n = n^2 - 6n + 11, \quad n \geq 1.$$

Given that the kth term of the sequence is 38, find the value of k. **(3)**

4. *(a)* Evaluate

$$(36^{\frac{1}{2}} + 16^{\frac{1}{4}})^{\frac{1}{3}}.$$ **(3)**

(b) Solve the equation

$$3x^{-\frac{1}{2}} - 4 = 0.$$ **(3)**

6. $$f(x) = x^2 - 10x + 17.$$

(a) Express f(x) in the form $a(x + b)^2 + c$. **(3)**

(b) State the coordinates of the minimum point of the curve $y = f(x)$. **(1)**

(c) Deduce the coordinates of the minimum point of each of the following curves:

(i) $y = f(x) + 4$,

(ii) $y = f(2x)$. **(4)**

8. *(a)* The first and third terms of an arithmetic series are 3 and 27 respectively.

(i) Find the common difference of the series.

(ii) Find the sum of the first 11 terms of the series. **(5)**

(b) Find the sum of the integers between 50 and 150 which are divisible by 8. **(5)**

10.

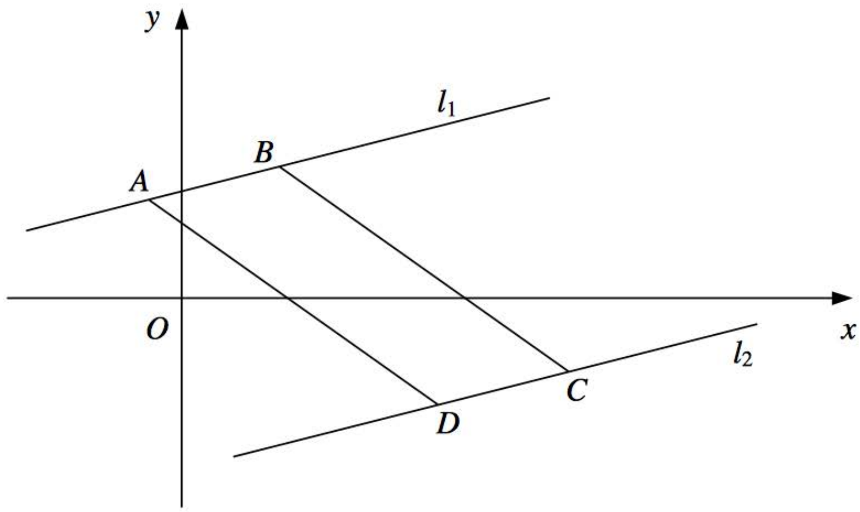

Figure 1

Figure 1 shows the parallelogram *ABCD*.

The points *A* and *B* have coordinates $(-1, 3)$ and $(3, 4)$ respectively and lie on the straight line l_1.

(a) Find an equation for l_1, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

The points *C* and *D* lie on the straight line l_2 which has the equation $x - 4y - 21 = 0$.

(b) Show that the distance between l_1 and l_2 is $k\sqrt{17}$, where k is an integer to be found. **(7)**

(c) Find the area of parallelogram *ABCD*. **(2)**

2. Express $\sqrt{22.5}$ in the form $k\sqrt{10}$. **(4)**

4. *(a)* Solve the inequality

$$4(x - 2) < 2x + 5.$$ **(2)**

(b) Find the value of y such that

$$4^{y+1} = 8^{2y-1}.$$ **(4)**

7. As part of a new training programme, Habib decides to do sit-ups every day.

He plans to do 20 per day in the first week, 22 per day in the second week, 24 per day in the third week and so on, increasing the daily number of sit-ups by two at the start of each week.

(a) Find the number of sit-ups that Habib will do in the fifth week. **(3)**

(b) Show that he will do a total of 1512 sit-ups during the first eight weeks. **(2)**

In the nth week of training, the number of sit-ups that Habib does is greater than 300 for the first time.

(c) Find the value of n. **(3)**

8. Some ink is poured onto a piece of cloth forming a stain that then spreads.

The area of the stain, A cm^2, after t seconds is given by

$$A = (p + qt)^2,$$

where p and q are positive constants.

Given that when $t = 0$, $A = 4$ and that when $t = 5$, $A = 9$,

(a) find the value of p and show that $q = \frac{1}{5}$, **(5)**

(b) find $\frac{\mathrm{d}A}{\mathrm{d}t}$ in terms of t, **(4)**

(c) find the rate at which the area of the stain is increasing when $t = 15$. **(2)**

3.

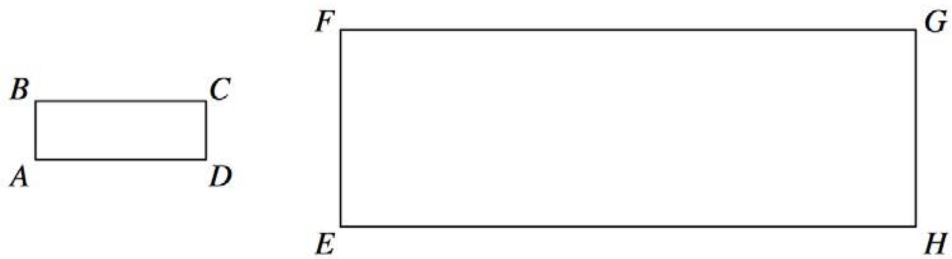

Figure 1

Figure 1 shows the rectangles *ABCD* and *EFGH* which are similar.

Given that $AB = (3 - \sqrt{5})$ cm, $AD = \sqrt{5}$ cm and $EF = (1 + \sqrt{5})$ cm, find the length *EH* in cm, giving your answer in the form $a + b\sqrt{5}$ where *a* and *b* are integers. **(6)**

5. *(a)* By completing the square, find in terms of the constant k the roots of the equation

$$x^2 + 2kx + 4 = 0.$$ **(4)**

(b) Hence find the exact roots of the equation

$$x^2 + 6x + 4 = 0.$$ **(2)**

6. *(a)* Evaluate

$$\sum_{r=1}^{50} (80 - 3r). \qquad \textbf{(3)}$$

(b) Show that

$$\sum_{r=1}^{n} \frac{r+3}{2} = kn(n+7),$$

where k is a rational constant to be found. **(4)**

10. The straight line l has gradient 3 and passes through the point $A\ (-6, 4)$.

(a) Find an equation for l in the form $y = mx + c$. **(2)**

The straight line m has the equation $x - 7y + 14 = 0$.

Given that m crosses the y-axis at the point B and intersects l at the point C,

(b) find the coordinates of B and C, **(4)**

(c) show that $\angle BAC = 90°$, **(4)**

(d) find the area of triangle ABC. **(4)**

StudentGrounds

Solomon L

2. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{u_n + 1}{3}, \quad n = 1, 2, 3, \ldots$$

Given that $u_3 = 5$,

(a) find the value of u_4, **(1)**

(b) find the value of u_1. **(3)**

4. Find the set of values of x for which

(a) $6x - 11 > x + 4$, **(2)**

(b) $x^2 - 6x - 16 < 0$, **(3)**

(c) both $6x - 11 > x + 4$ and $x^2 - 6x - 16 < 0$. **(1)**

7. Given that

$$f'(x) = 5 + \frac{4}{x^2}, \quad x \neq 0,$$

(a) find an expression for f(x). **(3)**

Given also that

$$f(2) = 2f(1),$$

(b) find f(4). **(5)**

9. The first two terms of an arithmetic series are $(t - 1)$ and $(t^2 - 5)$ respectively, where t is a positive constant.

(a) Find and simplify expressions in terms of t for

(i) the common difference of the series,

(ii) the third term of the series. **(4)**

Given also that the third term of the series is 19,

(b) find the value of t, **(2)**

(c) show that the 10th term of the series is 75, **(3)**

(d) find the sum of the first 40 terms of the series. **(2)**