

# Simultaneous Equations (ch 3 & 4)

1.  $y = 2x^2 - 3x + 4$ ,  $y = 4x + 1$

~~4x+1~~  $4x + 1 = 2x^2 - 3x + 4$

$$2x^2 - 7x + 3 = 0$$

$$(2x-1)(x-3) = 0$$

$$x = \frac{1}{2} \text{ or } x = 3$$

$$y = 3 \quad y = 13$$

2.  $x + y = 2$ ,  $x^2 + 2y^2 = 11$

$$x = 2 - y$$

$$(2-y)^2 + 2y^2 = 11$$

$$4 - 4y + y^2 + 2y^2 = 11$$

$$3y^2 - 4y - 7 = 0$$

$$(3y-7)(y+1) = 0$$

$$y = \frac{7}{3} \text{ or } y = -1$$

$$x = -\frac{1}{3} \quad x = 3$$

3.  $x^2 + 5x + y = 4$ ,  $x + y = 8$   
 $y = 8 - x$

$$x^2 + 5x + 8 - x = 4$$

$$x^2 + 4x + 4 = 0$$

$$(x+2)(x+2) = 0$$

$$x = -2$$

$$y = 10 \quad (\text{one repeated root})$$

4.  $x^2 + y^2 = 25$ ,  $2x + y - 5 = 0$   
 $y = 5 - 2x$

$$x^2 + (5-2x)^2 = 25$$

$$x^2 + 25 - 20x + 4x^2 - 25 = 0$$

$$5x^2 - 20x = 0$$

$$5x(x-4) = 0$$

$$x = 0 \text{ or } x = 4$$

$$y = 5 \quad y = -3$$

5.  $y - x = 4$ ,  $2x^2 + xy + y^2 = 8$

(i)  $y = x + 4$

$$2x^2 + x(x+4) + (x+4)^2 = 8 = 0$$

$$2x^2 + x^2 + 4x + x^2 + 8x + 16 - 8 = 0$$

$$4x^2 + 12x + 8 = 0$$

$$x^2 + 3x + 2 = 0$$

(ii)  $(x+2)(x+1) = 0$

$$x = -2, x = -1$$

$$6. (i) y = 2x + 2, \quad y = x^2 + 3x - 18$$

$$2x + 2 = x^2 + 3x - 18$$

$$x^2 + x - 20 = 0$$

$$(x + 5)(x - 4) = 0$$

$$x = -5 \quad \text{or} \quad x = 4$$

$$y = -8$$

$$y = 10$$

$$(ii) y = 2x - 20, \quad y = x^2 + 3x - 18$$

$$2x - 20 = x^2 + 3x - 18$$

$$x^2 + x + 2 = 0$$

now find the discriminant:

$$b^2 - 4ac = 1 - 4 \times 1 \times 2 = -7 < 0 \quad \therefore \text{no real solutions.}$$

$$(iii) y = 2x + k, \quad y = x^2 + 3x - 18$$

$$2x + k = x^2 + 3x - 18$$

$$x^2 + x - 18 - k = 0$$

$$\text{One solution} \Rightarrow b^2 - 4ac = 0$$

$$1 - 4 \times 1 \times (-18 - k) = 0$$

$$1 + 72 + 4k = 0$$

$$4k = -73$$

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$$k = -\frac{73}{4} = -18\frac{1}{4}$$