

The Discriminant (ch 4)

1. (i) $3x^2 - 4x + 2$
 $b^2 - 4ac = 16 - 4 \times 3 \times 2 = -8$

(ii) no real roots.

2. (i) $-x^2 + 4x - 7$
 $b^2 - 4ac = 16 - 4 \times 1 \times -7$
 $= 16 - 28$
 $= -12$

(ii) no real roots.

The graph does not cross the x -axis, therefore as it is a negative quadratic the graph must be below the x -axis, and $-x^2 + 4x - 7 < 0$.

3. (i) $2x^2 + 6x + 7$
 $b^2 - 4ac = 36 - 4 \times 2 \times 7$
 $= 36 - 56$
 $= -20$

(ii) no real roots.

The graph does not cross the x -axis. As it is a positive quadratic it must be above the x -axis and $2x^2 + 6x + 7 > 0$.

(b) $kx^2 + (4k+1)x + (3k+1) = 0$

Repeated root $\Rightarrow b^2 - 4ac = 0$

$$(4k+1)^2 - 4k(3k+1) = 0$$

$$16k^2 + 8k + 1 - 12k^2 - 4k = 0$$

$$4k^2 + 4k + 1 = 0$$

$$(2k+1)(2k+1) = 0$$

$$k = -\frac{1}{2}$$

4. (i) $3x^2 + 5x + 8$
 $b^2 - 4ac = 25 - 4 \times 3 \times 8$
 $= 25 - 96$
 $= -71$

(ii) No points of intersection with the x -axis.

5. $x^2 + 4kx + 3k = 0$

(i) Distinct real roots $\Rightarrow b^2 - 4ac > 0$

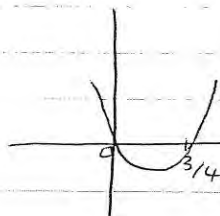
$$(4k)^2 - 4 \times 1 \times 3k > 0$$

$$16k^2 - 12k > 0$$

$$4k^2 - 3k > 0$$

$$k(4k - 3) > 0$$

(ii) $k < 0$ or $k > \frac{3}{4}$



(iii) Tangent \Rightarrow one repeated root $\Rightarrow b^2 - 4ac = 0$

$$k(4k-3) = 0$$

$$k = 0 \text{ or } k = 3/4$$

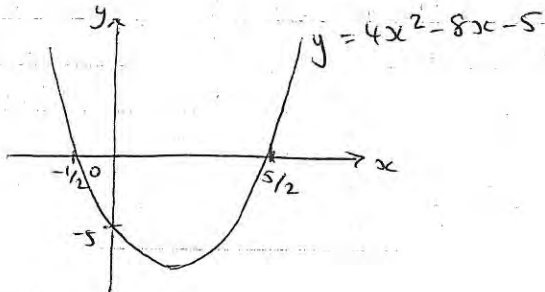
6. (i) $y = 4x^2 - 8x - 5$

$$4x^2 - 8x - 5 = 0$$

$$(2x + 1)(2x - 5) = 0$$

$$x = -1/2 \text{ or } x = 5/2$$

(ii)



(iii) $4x^2 - 8x - 5 = k$

one solution $\Rightarrow b^2 - 4ac = 0$

$$8^2 - 4 \times 4 \times (-5 - k) = 0$$

$$64 + 80 + 16k = 0$$

$$16k = -144$$

$$k = -9$$