Differentiation (ch 5)

1. $y = x^3 + x^{1/2}$

$$\frac{dy}{dx} = 3x^2 + \tfrac{1}{2}x^{-1/2}$$

2. $f(x) = 3x^2 - 5x - 2$

$$f'(x) = 6x - 5$$

3. $f(x) = 2x^3 - x^2 + x^{3/2} + 5$

(i) $f'(x) = 6x^2 - 2x + \tfrac{3}{2}x^{1/2}$

(ii) when $x = 4$,

$$\begin{aligned} f'(4) &= 6 \times 16 - 8 + \tfrac{3}{2}\sqrt{4} \\ &= 96 - 8 + 3 \\ &= 91 \end{aligned}$$

4.(i) A $(2, 2^3 + 3) = (2, 11)$

B $(2{\cdot}01, 2{\cdot}01^3 + 3) = (2{\cdot}01, 11{\cdot}120601)$

C $(2{\cdot}1, 2{\cdot}1^3 + 3) = (2{\cdot}1, 12{\cdot}261)$

gradient of AC $= \dfrac{12{\cdot}261 - 11}{2{\cdot}1 - 2} = \dfrac{1{\cdot}261}{0{\cdot}1} = 12{\cdot}61$

(ii) gradient of AB $= \dfrac{11.120601 - 11}{2.01 - 2} = \dfrac{0.120601}{0.01} = 12.0601$

(iii) At A, $\dfrac{dy}{dx} = 3x^2$,

$x = 2$, $\therefore$ gradient at A $= 3 \times 2^2 = 12$.

As the points get closer to A,
The gradients of the chords will get closer and closer to the gradient of the tangent to the curve at A, which is 12.

5. (i) $y = x(x^2+4) = x^3 + 4x$

$\dfrac{dy}{dx} = 3x^2 + 4$

(ii) $x = 1$; $\dfrac{dy}{dx} = 3 \times 1^2 + 4 = 7$

$y = 5$

$y - 5 = 7(x-1)$

$y = 7x - 2$

(iii) $y = x^3 + ax$

$\dfrac{dy}{dx} = 3x^2 + a$

Gradient of normal $= 1/2$ $\therefore$ gradient of tangent $= -2$ (when $x = 2$, $y = b$)

$3x^2 + a = -2$.

$3 \times 2^2 + a = -2$

$a = -14$

$y = x^3 + ax$

$b = 2^3 - 14 \times 2$

$= 8 - 28$

$= -20$

6. (i) $y = x^2 + 4x + 3$

$\dfrac{dy}{dx} = 2x + 4$ $\qquad x = 1$, $y = 8$

$m = 6$.

$y - 8 = 6(x-1)$

$y = 6x + 2$

(ii) $y = 4x + k$ tangent to $y = x^2 + 4x + 3$ at P.

$\dfrac{dy}{dx} = 2x + 4$

$2x + 4 = 4$

$x = 0$, $y = 3$.

$y = 4x + k$

$3 = 0 + k$

$\underline{k = 3}$