Paper Reference(s)

# 6663/01

# Edexcel GCE

## Core Mathematics C1

## Advanced Subsidiary

# Sequences and Series

**Calculators may NOT be used for these questions.**

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' might be needed for some questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 9 questions in this test.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear.

Answers without working may not gain full credit.

**1.** A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3}, \qquad n \geq 1,$$

$$a_1 = 2$$

(a) Find $a_2$ and $a_3$, leaving your answers in surd form.

**(2)**

(b) Show that $a_5 = 4$

**(2)**
**(Total 4 marks)**

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**2.** A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geq 1,$$

where $k$ is a constant.

(a) Write down an expression for $a_2$ in terms of $k$.

**(1)**

(b) Show that $a_3 = 4k - 21$.

**(2)**

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of $k$.

**(4)**
**(Total 7 marks)**

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**3.** A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, n > 1,$$

where $a$ is a constant.

(a) Find an expression for $x_2$ in terms of $a$.

**(1)**

(b) Show that $x_3 = a_2 - 3a - 3$.

**(2)**

Given that $x_3 = 7$,

(c) find the possible values of $a$.

**(3)**
**(Total 6 marks)**

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**4.** A sequence is given by:

$$x_1 = 1,$$

$$x_{n+1} = x_n(p + x_n),$$

where $p$ is a constant ($p \neq 0$).

(a) Find $x_2$ in terms of $p$.

**(1)**

(b) Show that $x_3 = 1 + 3p + 2p^2$.

**(2)**

Given that $x_3 = 1$,

(c) find the value of $p$,

**(3)**

(d) write down the value of $x_{2008}$.

**(2)**
**(Total 8 marks)**

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**5.** A sequence $a_1, a_2, a_3$ ... , is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5, \qquad n \geq 1,$$

where $k$ is a positive integer.

(a) Write down an expression for $a_2$ in terms of $k$.

**(1)**

(b) Show that $a_3 = 9k + 20$.

**(2)**

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of $k$.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10.

**(4)**

**(Total 7 marks)**

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**6.** A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = 3a_n - 5, \quad n \geq 1.$$

(a) Find the value of $a_2$ and the value of $a_3$.

**(2)**

(b) Calculate the value of $\sum_{r=1}^{5} a_r$

**(3)**
**(Total 5 marks)**

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**7.** The sequence $u_1, u_2, u_3, \ldots$ , is defined by the recurrence relation

$$u_{n+1} = (-1)^n u_n + d, \quad u_1 = 2, \quad \text{where } d \text{ is a constant.}$$

(a) Show that $u_5 = 2$.

**(4)**

(b) Deduce an expression for $u_{10}$, in terms of $d$.

**(1)**

Given that $u_3 = 3u_2$,

(c) find the value of $d$.

**(2)**
**(Total 7 marks)**

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**8.** A sequence is defined by the recurrence relation

$$u_{n+1} = \sqrt{\left(\frac{u_n}{2} + \frac{a}{u_n}\right)}, \quad n = 1, 2, 3, ...,$$

where $a$ is a constant.

(a) Given that $a = 20$ and $u_1 = 3$, find the values of $u_2$, $u_3$ and $u_4$, giving your answers to 2 decimal places.

**(3)**

(b) Given instead that $u_1 = u_2 = 3$,

(i) calculate the value of $a$,

**(3)**

(ii) write down the value of $u_5$.

**(1)**
**(Total 7 marks)**

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**9.** A sequence $a_1, a_2, a_3, ...$ is defined by

$$a_1 = k, \quad a_{n+1} = 4a_n - 7,$$

where $k$ is a constant.

(a) Write down an expression for $a_2$ in terms of $k$.

**(1)**

(b) Find $a_3$ in terms of $k$, simplifying your answer.

**(2)**

Given that $a_3 = 13$,

(c) find the value of $k$.

**(2)**
**(Total 5 marks)**

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| | | | | |
|---|---|---|---|---|
| **1.** | (a) | $a_2 = \left(\sqrt{4+3}\right) = \sqrt{7}$ | B1 | |
| | | $a_3 = \sqrt{\text{"their 7"}+3} = \sqrt{10}$ | B1ft | 2 |

**Note**

1st B1 for $\sqrt{7}$ only

2nd B1ft follow through their "7" in correct formula provided they have $\sqrt{n}$, where $n$ is an integer.

| | | | | |
|---|---|---|---|---|
| | (b) | $a_4 = \sqrt{10+3}\left(= \sqrt{13}\right)$ | M1 | |
| | | $a_5 = \sqrt{13+3} = 4$ * | A1 cso | 2 |

**Note**

M1 for an attempt to find $a_4$. Should see $\sqrt{\text{"}their\text{"}(a_3)^2+3}$.
Must see evidence for M1.

$a_4 = \sqrt{13}$ provided this follows from their $a_3$ working or answer is sufficient

A1cso for a correct solution (M1 explicit) must include the = 4.
Ending at $\sqrt{16}$ only is A0 and ending with $\pm$ 4 is A0.
Ignore any incorrect statements that are not used e.g. common difference = $\sqrt{3}$

Listing: A full list: 2 (=$\sqrt{4}$), $\sqrt{7}$, $\sqrt{10}$, $\sqrt{13}$, $\sqrt{16}$ = 4 is fine for M1A1

**ALT**

Formula: Some may state (or use) $a_n = \sqrt{3n+1}$ leading to $a_5 = \sqrt{3\times5+1} = 4$.

This will get marks in (a) [if correct values are seen] and can score the M1 in (b) if $a_n = \sqrt{3n+1}$ or $a_4 = \sqrt{13}$ are seen.

$\pm\sqrt{\ }$

If $\pm\sqrt{\ }$ appear any where ignore in part (a) and withhold the final A mark only

**[4]**

**2.** (a) $(a_2 =) 2k - 7$ B1 1

(b) $(a_3 =) 2(2k - 7) - 7$ or $4k - 14 - 7$, $= 4k - 21$ (*) M1, A1cso 2

**Note**

M1 must see 2(their $a_2$) – 7 or $2(2k - 7) - 7$ or $4k - 14 - 7$. Their $a_2$ must be a function of $k$.

A1cso must see the $2(2k - 7) - 7$ or $4k - 14 - 7$ expression and the $4k - 21$ with no incorrect working

(c) $(a_4 =) 2(4k - 21) - 7$ $(= 8k - 49)$ M1

$\sum_{r=1}^{4} a_r = k +$ "$(2k - 7)$" $+ (4k - 21) +$ "$(8k - 49)$" M1

$k + (2k - 7) + (4k - 21) + (8k - 49) = 15k - 77 = 43$ $k = 8$ M1 A1 4

**Note**

1st M1 for an attempt to find $a_4$ using the given rule. Can be awarded for $8k - 49$ seen.

**Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks.**

2nd M1 for attempting the sum of the 1st 4 terms. Must have "+" not just, or clear attempt to sum.

Follow through their $a_2$ and $a_4$ provided they are linear functions of $k$.

Must lead to linear expression in $k$. Condone use of their linear $a_3 \neq 4k - 21$ here too.

3rd M1 for forming a linear equation in $k$ using their sum and the 43 and attempt to solve for $k$ as far as $pk = q$

A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0

Answer Only (e.g. trial improvement)

Accept $k = 8$ only if $8 + 9 + 11 + 15 = 43$ is seen as well

Sum $\underline{a_2 + a_3 + a_4 + a_5}$ or $\underline{a_2 + a_3 + a_4 +}$

Allow: M1 if $8k - 49$ is seen, M0 for the sum (since they are not adding the 1st 4 terms) then M1 if they use their sum along with the 43 to form a linear equation and attempt to solve but A0

**[7]**

| | | | | |
|---|---|---|---|---|
| **3.** | (a) | $[x_2 =]\ a - 3$ | B1 | 1 |

B1 for $a \times 1 - 3$ or better. Give for $a - 3$ in part (a) or if it appears in (b) they must state $x_2 = a - 3$
This must be seen in (a) or before the $a(a - 3) - 3$ step.

| | | | |
|---|---|---|---|
| (b) | $[x_3 = ]\ ax_2 - 3$ or $a(a - 3) - 3$ | M1 | |
| | $= a(a - 3) - 3$ (*) | | |
| | $= a^2 - 3a - 3$ (*) | A1cso | 2 |

(*) both lines needed for A1

M1 for clear show that. Usually for $a(a - 3) - 3$ but can follow through their $x_2$ and even allow $ax_2 - 3$

A1 for correct processing leading to printed answer. Both lines needed and no incorrect working seen.

| | | | |
|---|---|---|---|
| (c) | $a^2 - 3a - 3 = 7$ | | |
| | $a^2 - 3a - 10 = 0$ or $a^2 - 3a = 10$ | M1 | |
| | $(a - 5)(a + 2) = 0$ | dM1 | |
| | $\underline{a = 5 \text{ or } -2}$ | A1 | 3 |

1st M1 for attempt to form a correct equation and start to collect terms. It must be a quadratic but need not lead to a 3TQ = 0

2nd dM1 This mark is dependent upon the first M1. for attempt to factorize their 3TQ = 0 or to solve their 3TQ = 0. The "=0"can be implied.

$(x \pm p)(x \pm q) = 0$, where $pq = 10$ or $(x \pm \frac{3}{2})^2 \pm \frac{9}{4} - 10 = 0$ or correct use of quadratic formula with ±

They must have a form that leads directly to 2 values for $a$. Trial and Improvement that leads to only one answer gets M0 here.

A1 for both correct answers. Allow $x =$…

Give 3/3 for correct answers with no working or trial and improvement that gives both values for $a$

**[6]**

| | | | | |
|---|---|---|---|---|
| **4.** | (a) | $1(p + 1)$ or $p + 1$ | B1 | 1 |
| | (b) | $((a))(p + (a))$ [(a) must be a function of $p$]. $[(p+1)(p+p+1)]$ | M1 | |
| | | $=1+3p+2p^2$ (*) | A1cso | 2 |

M: Valid attempt to use the given recurrence relation to find $x_3$.
Missing brackets, e.g. $p + 1(p + p + 1)$ Condone for the M1, then if all terms in the expansion are correct, with working fully shown, M1 A1 is still allowed.

Beware 'working back from the answer',
e.g. $1 + 3p + 2p^2 = (1 + p)(1 + 2p)$ scores no marks unless the recurrence relation is justified.

| | | | | |
|---|---|---|---|---|
| | (c) | $1+3p+2p^2=1$ | M1 | |
| | | $p(2p+3)=0 \quad p = ...$ | M1 | |
| | | $p = -\frac{3}{2}$ (ignore $p = 0$, if seen, even if 'chosen' as the answer) | A1 | 3 |

2nd M: Attempt to solve a quadratic equation in $p$ (e.g. quadratic formula or completing the square). The equation must be based on $x_3 = 1$.

The attempt must lead to a non-zero solution, so just stating the zero solution $p = 0$ is M0.

A: The A mark is dependent on both M marks.

| | | | | |
|---|---|---|---|---|
| | (d) | Noting that even terms are the same. | M1 | |

This M mark can be implied by listing at least 4 terms, e.g. $1, -\frac{1}{2}, 1, -\frac{1}{2}, ...$

| | | |
|---|---|---|
| $x_{2008} = -\frac{1}{2}$ | A1 | 2 |

M: Can be implied by a correct answer for their $p$ (answer is $p + 1$), and can also be implied if the working is 'obscure').

Trivialising, e.g. $p = 0$, so every term = 1, is M0.

If the additional answer $x_{2008} = 1$ (from $p = 0$) is seen, ignore this (isw).

**[8]**

| | | | | |
|---|---|---|---|---|
| **5.** | (a) | ($a_2$ =)<u>$3k + 5$</u> [must be seen in part (a) or labelled $a_2$ =] | B1 | 1 |
| | (b) | ($a_3$ =)$3(3k + 5) + 5$ | M1 | |
| | | = <u>$9k + 20$</u> (*) | A1cso | 2 |

M1 for attempting to find $a_3$, follow through their $a_2 \neq k$.

A1cso for simplifying to printed result with no incorrect working seen.

| | | | | |
|---|---|---|---|---|
| (c) | (i) | $a_4 = 3(9k + 20) + 5$ (= $27k + 65$) | M1 | |
| | | $\sum_{r=1}^{4} a_r = k + (3k + 5) + (9k + 20) + (27k + 65)$ | M1 | |
| | (ii) | $= 40k + 90$ | A1 | |
| | | = <u>$10(4k + 9)$</u> (or explain why divisible by 10) | A1ft | 4 |

1st M1 for attempting to find $a_4$. Can allow a slip here e.g. $3(9k + 20)$ [i.e. forgot +5]

2nd M1 for attempting sum of 4 relevant terms, follow through their (a) and (b).
Must have 4 terms starting with $k$.
Use of arithmetic series formulae at this point is M0A0A0

1st A1 for simplifying to $40k + 90$ or better

2nd A1ft for taking out a factor of 10 or dividing by 10 or an explanation in words true $\forall k$.
Follow through their sum of 4 terms provided that both Ms are scored and their sum <u>is</u> divisible by 10.
A comment is <u>not</u> required.
e.g. $\frac{40k+90}{10} = 4k + 9$ is OK for this final A1.

S.C. $\sum_{r=2}^{5} a_r = 120k + 290 = 10(12k + 29)$ can have M1M0A0A1ft.

**[7]**

| | | | | |
|---|---|---|---|---|
| **6.** | (a) | $a_2 = 4$ | B1 | |
| | | $a_3 = 3 \times a_2 - 5 = 7$ | B1f.t. | 2 |
| | | *2nd B1f.t. Follow through their $a_2$ but it must be a value. $3\times 4 - 5$ is B0. Give wherever it is first seen.* | | |
| | (b) | $a_4 = 3a_3 - 5(= 16)$ and $a_5 = 3a_4 - 5(= 43)$ | M1 | |
| | | $3 + 4 + 7 + 16 + 43$ | M1 | |
| | | $= 73$ | A1c.a.o. | 3 |
| | | *1st M1 For two further attempts to use of $a_{n+1} = 3a_n - 5$, wherever seen. Condone arithmetic slips* | | |
| | | *2nd M1 For attempting to add 5 relevant terms (i.e. terms derived from an attempt to use the recurrence formula) or an expression. Follow through their values for $a_2 - a_5$* | | |
| | | *Use of formulae for arithmetic series is M0A0 but could get 1st M1 if $a_4$ and $a_5$ are correctly attempted.* | | |
| | | | | **[5]** |
| **7.** | (a) | $u_2 = (-1)(2) + d = -2 + d$ | B1 | |
| | | $u_3 = (-1)^2(-2 + d) + d = -2 + 2d$ | M1 | |
| | | *Attempting to find $u_3$ in terms of $d$* | | |
| | | $u_4 = (-1)^3(-2 + 2d) + d = 2 - d$ | A1 | 4 |
| | | *$u_3$ and $u_4$ correct* | | |
| | | $u_5 = (-1)^4(2 - d) + d = 2$ (*) cso | | |
| | | *fully correct* | | |
| | (b) | $u_{10} = u_2 = d - 2$ o.e. | B1ft | 1 |
| | | *their $u_2$ must contain $d$* | | |
| | (c) | $-2 + 2d = 3(-2 + d) \Rightarrow d = 4$ | M1 A1 | 2 |
| | | *M1 equating their $u_3$ to their $3u_2$ must contain $d$* | | |
| | | | | **[7]** |

| | | | | |
|---|---|---|---|---|
| **8.** | (a) | $u_2 = \sqrt{\left(\frac{3}{2}+\frac{20}{3}\right)}$ = 2.85773…. = <u>2.86</u> | M1 | |
| | | $u_3$ = 2.90300… = <u>2.90</u> | A1 c.a.o | |
| | | $u_4$ = 2.88806…. = <u>2.89</u> | A1 c.a.o | 3 |
| | | S.C. [If $u_3$ = AWRT 2.90 and $u_4$ = AWRT 2.89 penalise once only] | | |
| | | *M1 Correct expression or AWRT 2.86* | | |
| | (b) | (i) $3 = \sqrt{\left(\frac{3}{2}+\frac{a}{3}\right)}$ or $9 = \frac{3}{2}+\frac{a}{3}$ | M1 | |
| | | $\frac{a}{3} = 9 - \frac{3}{2}$ or $a = 3\left(9-\frac{3}{2}\right)$ | M1 | |
| | | <u>$a = 22.5$</u> | A1 | 3 |
| | | *M1 A correct equation for a, with or without* $\sqrt{\ }$ *.* | | |
| | | *M1 Attempt correct manipulation to ka = ,(k > 0).* | | |
| | | (ii) (If $u_1 = u_2$, then $u_2 = u_3$……..) $u_5$ <u>= 3</u> | B1 | 1 |

**[7]**

| | | | | |
|---|---|---|---|---|
| **9.** | (a) | $4k - 7$ | B1 | |
| | (b) | $4(4k - 7) - 7 = 16k - 35$ | M1 A1 | 2 |
| | (c) | $16k - 35 = 13$ $\quad k = 3$ | M1 A1 | 2 |

**[5]**

**1.** This proved to be a straightforward question for most candidates who worked through it carefully and gained full marks. A few noticed that the numbers inside the square root formed an arithmetic sequence and this sometimes distracted them as they tried to use formulae for arithmetic series.

There were still some candidates who did not understand the notation and interpreted $a_n$ as $\sqrt{n^2 + 3}$.

Some were confused by the nested square roots and we saw $a_3 = \sqrt{7^2 + 3}$ and others thought $\left(\sqrt{7}\right)^2 = 49$ but overall this question was answered well.

**2.** There were far fewer cases of candidates not understanding how an inductive formula like this works and many were able to answer parts (a) and (b) successfully. Part (b) required the candidates to "show" a given result and most gave the expression $2(2k – 7) – 7$ which was fine but a small minority thought the pattern must be $2 \times 2k – 3 \times 7$. Part (c) met with mixed success: many found $a4$ but some then solved $a4 = 43$ whilst others assumed that the series was arithmetic and attempted to use a formula such as $\frac{4}{2}(a_1 + z_4)$. Those who did attempt the correct sum occasionally floundered with the arithmetic but there were plenty of fully correct solutions seen.

**3.** The notation associated with sequences given in this form still causes difficulties for some candidates and as a result parts (a) and (b) were often answered less well than part (c). A common error in the first two parts was to leave an $x$ in the expression but most of those who could handle the notation gave clear and accurate answers. There were the usual errors in part (c), with $a^2 – 3a – 4 = 0$ appearing quite often and it was encouraging to see most candidates factorising their quadratic expression confidently as a means of solving the equation. A few candidates still use a trial and improvement approach to questions of this type and they often stopped after finding just one solution and gained no credit.

**4.** Although just a few candidates failed to understand the idea of the recurrence relation, most managed to complete the first two parts successfully. A major concern in part (b), however, was the widespread lack of brackets in the algebraic expressions. It was usually possible for examiners to interpret candidates' intentions generously, but there needs to be a greater awareness that, for example, $1 + p(1 + 2p)$ is not an acceptable alternative to $(1 + p)(1 + 2p)$.

The given answer to part (b) enabled the vast majority of candidates to start part (c) correctly, but the main problem with this part was in solving $2p^2 + 3p = 0$, which proved surprisingly difficult for some. Attempts to complete the square usually failed, while the quadratic formula method, although generally more successful, often suffered from mistakes related to the fact that $c$ was zero. Those who did manage to factorise the expression sometimes gave the answer $p = \frac{3}{2}$ instead of $p = -\frac{3}{2}$. It was clear that candidates would have been much happier solving a 3-term quadratic equation. Those who trivialised the question by giving only the zero solution (despite the condition $p > 0$) scored no further marks in the question.

Part (d) proved challenging for many candidates. Some used the solution $p = 0$ and some tried to make use of the sum formula for an arithmetic series. Few candidates were successful, but those who wrote out the first few terms were more likely to spot the 'oscillatory' nature of the sequence. Good candidates stated that even terms were all equal to $-\frac{1}{2}$ and therefore the 2008th term was $-\frac{1}{2}$. Quite a large number of candidates were able to express $x_{2008}$ in terms of $x_{2007}$, but those who simply substituted 2007 into one of their expressions often wasted time in tedious arithmetic that led to a very large answer.

**5.** Many of the comments made on the June 2006 paper would apply here too. Many candidates were clearly not familiar with the notation and a number used arithmetic series formulae to find the sum in part (c) although this was less common than in June 2006.

Apart from those candidates who had little idea about this topic most were able to answer parts (a) and (b) correctly. In part (c) many attempted to find $a_4$ using the recurrence relation and those who were not tempted into using the arithmetic series formulae often went on to attempt the sum and usually obtained $40k + 90$ which they were easily able to show was divisible by 10.
Some lost marks for poor arithmetic $30k + 90$ and $40k + 80$ being some of the incorrect answers seen.

**6.** Most candidates knew how to start this question and full marks for part (a) was common, however some lost out due to poor arithmetic such as $3 \times 3 - 5 = 6 - 5 = 1$ for $a_2$ A minority of candidates though had no idea how to interpret the recurrence relation notation with a significant number interpreting $3a - 5$ as $3 \times n - 5$. In part (b) many candidates were convinced that the series had to be arithmetic and they gained no further marks. Some did find $a_4 + a_5$ and $a$ correctly but then used the arithmetic formula $\frac{n(a+l)}{2}$ with $l = 43$ Clearly students are familiar with the work on arithmetic series but in some cases this seems to have overshadowed their understanding of recurrence relations.

**7.** This question was poorly answered. Many candidates started by using $u_{n+1} = (-1)^{n+1} u_n + d$ or by establishing a value for $d$, usually by creating a $u_0$. The fact that all these also gave $u_5 = 2$ tended to lull candidates into a false sense of security. In part (b) most candidates realised $u_{10} = u_2$. In part (c) those who were not successful in (a) did on the whole recalculate $u_2$ and $u_3$, and then equate $u_3$ to $3u_2$ to gain the method mark

**8.** This question was answered very well and many scored full marks. In part (a) some failed to give answers to 2 decimal places and sometimes the wrong substitution was made or the square root omitted. Part (b)(ii) caused a few problems, some candidates did not appreciate the instruction to "write down" and worked through $u_3$, and $u_4$ before stating the answer. Others started with 2.89 instead of 3.

**9.** No Report available for this question.