Paper Reference(s)

6663/01 Edexcel GCE Core Mathematics C1 Advanced Subsidiary

Sequences and Series: Arithmetic Series

Calculators may NOT be used for these questions.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' might be needed for some questions.

The marks for the parts of questions are shown in round brackets, e.g. (2).

There are 18 questions in this test.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear.

Answers without working may not gain full credit.

1. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays $\pounds a$ for their first day, $\pounds(a + d)$ for their second day, $\pounds(a + 2d)$ for their third day, and so on, thus increasing the daily payment by $\pounds d$ for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

| (a) | Use this information to form an equation in <i>a</i> and <i>d</i> . | (2) |
|------|---|------------------------|
| A pi | cker who works for all 30 days will earn a total of £1005 | |
| (b) | Show that $15(a + 40.75) = 1005$ | (2) |
| (c) | Hence find the value of <i>a</i> and the value of <i>d</i> . | (4) (Total 8 marks) |
| | | |

2. Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and so on, so that the amounts of money she gave each year formed an arithmetic sequence.

| (a) Find the amount of money she | gave in Year 10. |
|----------------------------------|------------------|
|----------------------------------|------------------|

(b) Calculate the total amount of money she gave over the 20-year period.

(3)

(2)

Kevin also gave money to the charity over the same 20-year period.

He gave $\pounds A$ in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference $\pounds 30$.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(c) Calculate the value of A.

(4) (Total 9 marks)

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3. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

- (a) the value of d, (3)
- (b) the value of *a*,
- (c) the total number of houses built in Oldtown over the 40-year period.

(3) (Total 8 marks)

(2)

(2)

(2)

(4)

4. The first term of an arithmetic series is *a* and the common difference is *d*.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

- (a) Use this information to write down two equations for *a* and *d*.
- (b) Show that a = -7.5 and find the value of d.

The sum of the first n terms of the series is 2750.

(c) Show that *n* is given by

$$n^2 - 15n = 55 \times 40.$$

(d) Hence find the value of *n*.

(3) (Total 11 marks)

| Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km. | | | | |
|---|--|-----|--|--|
| (a) | Show that on the 4th Saturday of training she runs 11 km. | (1) | | |
| (b) | Find an expression, in terms of n , for the length of her training run on the n th Saturday. | | | |
| (c) | Show that the total distance she runs on Saturdays in <i>n</i> weeks of training is $n(n + 4)$ km. | (2) | | |
| | | (3) | | |
| On t | he <i>n</i> th Saturday Sue runs 43 km. | | | |
| (d) | Find the value of <i>n</i> . | (2) | | |
| (e) | Find the total distance, in km, Sue runs on Saturdays in <i>n</i> weeks of training. | (2) | | |
| | (Total 10 m | | | |
| The | first term of an arithmetic sequence is 30 and the common difference is -1.5 | | | |
| (a) | Find the value of the 25th term. | (2) | | |
| The | <i>r</i> th term of the sequence is 0. | | | |
| (b) | Find the value of <i>r</i> . | (2) | | |
| The | sum of the first <i>n</i> terms of the sequence is S_n . | | | |
| | | | | |

(3) (Total 7 marks)

- 7. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.
 - (a) Find the amount she saves in Week 200.

(3)

(b) Calculate her total savings over the complete 200 week period.

(3) (Total 6 marks)

8. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

| Row 1 | 0 |
|-------|----|
| Row 2 | CO |
| Row 3 | |

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of *n*, for the number of sticks required to make a similar arrangement of *n* squares in the *n*th row.

(3)

(3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(b) Find the total number of sticks Ann uses in making these 10 rows.

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the (k+1)th row,

(c) show that *k* satisfies (3k - 100)(k + 35) < 0.

(4)

(d) Find the value of k.

(2) (Total 12 marks)

9. An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day, he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

Find the value of *a* and the value of *d*.

(Total 7 marks)

| al | | lice's 11th birthday she started to receive an annual allowance. The first annual vance was £500 and on each following birthday the allowance was increased by | |
|----|----|--|--|
| (a | a) | Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200. | |
| (t | b) | Find the amount of Alice's annual allowance on her 18th birthday. | |
| (0 | c) | Find the total of the allowances that Alice had received up to and including her 18th birthday. | |

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(d) Find how old Alice was when she received her last allowance.

(7) (Total 13 marks)

- **11.** An arithmetic series has first term *a* and common difference *d*.
 - (a) Prove that the sum of the first *n* terms of the series is

$$\frac{1}{2}n[2a+(n-1)d].$$
(4)

The *r*th term of a sequence is (5r - 2).

(b) Write down the first, second and third terms of this sequence.

(1)

(c) Show that
$$\sum_{r=1}^{n} (5r-2) = \frac{1}{2}n(5n+1).$$
 (3)

(d) Hence, or otherwise, find the value of
$$\sum_{r=5}^{200} (5r-2)$$
.

(4) (Total 12 marks)

- 12. An arithmetic series has first term *a* and common difference *d*.
 - (a) Prove that the sum of the first *n* terms of the series is

$$\frac{1}{2}n[2a+(n-1)d].$$
(4)

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the *n*th month, where n > 21.

(b) Find the amount Sean repays in the 21st month.

(2)

(3)

Over the *n* months, he repays a total of $\pounds 5000$.

(c) Form an equation in *n*, and show that your equation may be written as

$$n^2 - 150n + 5000 = 0. \tag{3}$$

- (d) Solve the equation in part (c).
- (e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible solution to the repayment problem.

(1) (Total 13 marks)

C1 Sequences and series: Arithmetic series – Questions

- **13.** The *r*th term of an arithmetic series is (2r 5).
 - (a) Write down the first three terms of this series.
 - (b) State the value of the common difference.
 - (c) Show that $\sum_{r=1}^{n} (2r-5) = n(n-4)$.

(3) (Total 6 marks)

(2)

(1)

14. The first three terms of an arithmetic series are p, 5p - 8, and 3p + 8 respectively.

| (a) | Show that $p = 4$. | (2) |
|-----|--|------------------------|
| (b) | Find the value of the 40th term of this series. | (3) |
| (c) | Prove that the sum of the first n terms of the series is a perfect square. | |
| | | (3) (Total 8 marks) |

15. The sum of an arithmetic series is

$$\sum_{r=1}^{n} (80 - 3r)$$

(a) Write down the first two terms of the series.

(b) Find the common difference of the series.

Given that n = 50,

(c) find the sum of the series.

(3) (Total 6 marks)

(2)

(1)

16. In the first month after opening, a mobile phone shop sold 280 phones. A model for future trading assumes that sales will increase by x phones per month for the next 35 months, so that (280 + x) phones will be sold in the second month, (280 + 2x) in the third month, and so on.

Using this model with x = 5, calculate

- (a) (i) the number of phones sold in the 36th month,
 (2)
 (ii) the total number of phones sold over the 36 months.
 (2)
 The shop sets a sales target of 17 000 phones to be sold over the 36 months.
 Using the same model,
 (b) find the least value of *x* required to achieve this target.
 - (Total 8 marks)
- 17. (a) An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of this series is

$$\frac{1}{2}n[2a+(n-1)d].$$

(4)

The first three terms of an arithmetic series are k, 7.5 and k + 7 respectively.

- (b) Find the value of k. (2)
- (c) Find the sum of the first 31 terms of this series.

(4) (Total 10 marks)

- 18. An arithmetic series has first term *a* and common difference *d*.
 - (a) Prove that the sum of the first *n* terms of the series is

$$\frac{1}{2}n[2a+(n-1)d].$$
(4)

A polygon has 16 sides. The lengths of the sides of the polygon, starting with the shortest side, form an arithmetic sequence with common difference d cm.

The longest side of the polygon has length 6 cm and the perimeter of the polygon is 72 cm.

Find

(b) the length of the shortest side of the polygon,

(c) the value of d.

(5)

(2) (Total 11 marks)

1. (a) a + 29d = 40.75 or a = 40.75 - 29d or 29d = 40.75 - a M1 A1 2

<u>Note</u>

Parts (b) and (c) may run together

- M1 for attempt to use a + (n-1)d with n = 30 to form an equation . So a + (30-1)d = any number is OK
- A1 as written. Must see 29*d* not just (30 1)d. Ignore any floating £ signs e.g. a + 29d = £40.75 is OK for M1A1 These two marks must be scored in (a). Some may omit (a) but get correct equation in (c) [or (b)] but we do not give the marks retrospectively.

(b)
$$(S_{30}) = \frac{30}{2}(a+l)$$
 or $\frac{30}{2}(a+40.75)$ or $\frac{30}{2}(2a+(30-1)d)$ or
15(2a+29d) M1

So
$$1005 = 15[a + 40.75]$$
 * A1 cso 2

<u>Note</u>

Parts (b) and (c) may run together

M1 for an attempt to use an S_n formula with n = 30.

Must see one of the printed forms. (*S*₃₀ = is not required)

A1cso for forming an equation with 1005 and S_n and simplifying to printed answer. Condone £ signs e.g. $15[a + \pounds 40.75] = 1005$ is OK for A1

(c)
$$67 = a + 40.75$$
 so $a = (\pounds) 26.25 \text{ or } 2625 \text{p or}$
 $26\frac{1}{4} \text{ NOT } \frac{105}{4}$
 $29d = 40.75 - 26.25$ M1 A1

= 14.5 so
$$\underline{d} = (\underline{\pounds})0.50 \text{ or } 0.5 \text{ or } 50 \underline{p} \text{ or } \frac{1}{2}$$
 A1 4

<u>Note</u>

- 1st M1 for an attempt to simplify the given linear equation for *a*. Correct processes. Must get to ka = ... or k = a + m i.e. one step (division or subtraction) from a = ... Commonly: 15a = 1005 - 611.25 (= 393.75)
- 1st A1 For a = 26.25 or 2625p or $26\frac{1}{4}$ NOT $\frac{105}{4}$ or any other fraction
- 2^{nd} M1 for correct attempt at a linear equation for *d*, follow through their *a* or equation in (a) Equation just has to be linear in *d*, they don't have to simplify to d = ...
- 2^{nd} A1 depends upon 2^{nd} M1 and use of correct *a*. Do not penalise a second time if there were minor arithmetic errors in finding a provided *a* = 26.25 (o.e.) is used.

M1 A1

2

Do not accept other fractions other than $\frac{1}{2}$

If answer is in pence a "p" must be seen.

Sim Equ

Use this scheme: 1st M1A1 for *a* and 2^{nd} M1A1 for *d*. Typically solving: 1005=30a + 435d and 40.75 = a + 29d. If they find *d* first then follow through use of their *d* when finding *a*.

[8]

2. (a)
$$a + 9d = 150 + 9 \times 10 = 240$$

<u>Note</u>

M: Using a + 9d with at least one of a = 150 and d = 10.

Being 'one off' (e.g. equivalent to a + 10d), scores M0.

Correct answer with no working scores both marks.

'Listing' and other methods

M: Listing terms (found by a correct method with at least one of a = 150 and d = 10), and picking the <u>10th</u> term. (There may be numerical slips).

(b)
$$\frac{1}{2}n\{2a+(n-1)d\}=\frac{20}{2}\{2\times150+19\times10\},=4900$$
 M1 A1, A1 3

<u>Note</u>

M: Attempting to use the correct sum formula to obtain S_{20} , with at least one of a = 150 and d = 10. If the wrong value of n or a or d is used, the M mark is only scored if the correct sum formula has been quoted.

1st A: Any fully correct numerical version.

'Listing' and other methods

M: Listing sums, or listing and adding terms (found by a correct method with at least one of a = 150 and d = 10), far enough to establish the required sum. (There may be numerical slips). Note: 20^{th} term is 340.

A2 (scored as A1 A1) for 4900 (clearly selected as the answer).

If no working (or no legitimate working) is seen, but the answer 4900 is given, allow one mark (scored as M1 A0 A0).

(c) Kevin:
$$\frac{1}{2}n\{2a+(n-1)d\}=\frac{20}{2}\{2A+19\times 30\}$$
 B1

Kevin's total =
$$2 \times 4900$$
" (or 4900 " = $2 \times \text{Kevin's total}$ M1

$$\frac{20}{2} \{2A + 19 \times 30\} = 2 \times "4900"$$
 A1ft

'Listing' and other methods

By trial and improvement:

Obtaining a value of A for which Kevin's total is twice Jill's total, or Jill's total is twice Kevin's (using Jill's total from (b)): M1 Obtaining a value of A for which Kevin's total is twice Jill's total (using Jill's total from (b)): A1ft Fully correct solutions then score the B1 and final A1.

The answer 205 with no working (or no legitimate working) scores no marks.

[9]

3. (a) a + 9d = 2400 a + 39d = 600 M1 -1800

$$d = \frac{-1800}{30}$$
 $d = -60$ (accept ± 60 for A1) M1 A1 3

<u>Note</u>

If the sequence is considered 'backwards', an equivalent solution may be given using d = 60 with a = 600 and l = 2940 for part (b). This can still score full marks. **Ignore labelling of (a) and (b)**

 1^{st} M1 for an attempt to use 2400 and 600 in a+(n - 1)d formula. Must use both values

i.e. need a + pd = 2400 and a + qd = 600 where p = 8 or 9 and q = 38 or 39 (any combination)

 2^{nd} M1 for an attempt to solve <u>their</u> 2 linear equations in *a* and *d* as far as d = ...

A1 for $d = \pm 60$. Condone correct equations leading to d = 60or a + 8d = 2400 and a + 38d = 600 leading to d = -60. They should get penalised in (b) and (c).

NB This is a "one off" ruling for A1. Usually an A mark must follow from their work.

ALT

1st M1 for (30*d*) = ± (2400 - 600)
2nd M1 for (*d* =) ±
$$\frac{(2400 - 600)}{30}$$

A1 for $d = \pm 60$

a + 9d = 600, a + 39d = 2400 only scores M0 BUT if they solve to find $d = \pm 60$ then use ALT scheme above.

(b)
$$a - 540 = 2400 \ a = 2940$$

M1 A1 2

<u>Note</u>

M1 for use of their *d* in a correct linear equation to find *a* leading to a = ...

A1 their *a* must be compatible with their *d* so d = 60 must have a = 600and d = -60, a = 2940

So for example they can have 2400 = a + 9(60) leading to a = ... for M1 but it scores A0

Any approach using a list scores M1A1 for a correct a but M0A0 otherwise

(c) Total

$$= \frac{1}{2}n\{2a + (n-1)d\} = \frac{1}{2} \times 40 \times (5880 + 39 \times -60) \text{ (ft values of } a \text{ and } d) \text{ M1 A1 ft}$$
$$= \frac{70800}{41} \text{ Comparison}$$

<u>Note</u>

M1 for use of a correct S_n formula with n = 40 and at least one of *a*, *d* or *l* correct or correct ft.

 1^{st} A1ft for use of a correct S40 formula and both *a*, *d* or *a*, *l* correct or correct follow through

ALT

Total
$$=\frac{1}{2}n\{a+l\}=\frac{1}{2}\times40\times(2940+600)$$
 (ft value of *a*) M1 A1ft
2nd A1 for 70800 only

- 4

| [8] |
|-----|
| |

4. (a)
$$a+17d = 25$$
 or equiv. (for 1st B1),
 $a+20d = 32.5$ or equiv. (for 2nd B1), B1, B1 2

<u>Note</u>

Alternative:

1st B1:
$$d = 2.5$$
 or equiv.or $d = \frac{32.5 - 25}{3}$, No method required,
but $a = -17.5$ must not be assumed.

2nd B1: Either a + 17d = 25 or a + 20d = 32.5 seen, or used with a value of d... or for 'listing terms' or similar methods, 'counting back' 17 (or 20) terms.

(b) Solving (Subtract) 3d = 7.5 so d = 2.5M1 $a = 32.5 - 20 \times 2.5$ so a = -17.5 (*) A1cso 2

Note

- M1: In main scheme: for a full method (allow numerical or sign slips) leading to solution for d or a without assuming a = -17.5In alternative scheme: for using a *d* value to find a value for *a*.
- A1: Finding correct values for both a and d (allowing equiv. fractions such as d = 15/6), with no incorrect working seen.

$$2750 = \frac{n}{2}[-35 + \frac{5}{2}(n-1)]$$
M1A1ft

$$\{ 4 \times 2750 = n(5n - 75) \}$$

$$4 \times 550 = n(n-15)$$
 M1

$$\underline{n^2 - 15n = 55 \times 40} \quad (*)$$
 A1cso 4

Note

(c)

In the main scheme, if the given *a* is used to find *d* from one of the equations, then allow M1A1 if both values are checked in the 2nd equation.

 1^{st} M1 for attempt to form equation with correct S_n formula and 2750, with values of a and d.

1^{st} A1ft for a correct equation following through their d.

2nd M1 for expanding and simplifying to a 3 term quadratic.

2nd A1 for correct working leading to printed result (no incorrect working seen).

$$n^{2} - 15n - 55 \times 40 = 0$$
 or $n^{2} - 15n - 2200 = 0$ M1
 $(n - 55)(n + 40) = 0$ $n = \dots$ M1

$$55(n+40) = 0$$
 $n = \dots$ M1

$$n = 55$$
 (ignore - 40) A1 3

Note

(d)

 1^{st} M1 forming the correct 3TQ = 0. Can condone missing "= 0" but all terms must be on one side. First M1 can be implied (perhaps seen in (c), but there must be an attempt at (d) for it to be scored). 2nd M1 for attempt to solve 3TQ, by factorisation, formula or completing the square (see general marking principles at end of scheme). If this mark is earned for the 'completing the square' method or if the factors are written down directly, the 1st M1 is given by implication. A1 for n = 55 dependent on both Ms. Ignore – 40 if seen. No working or 'trial and improvement' methods in (d) score all 3

marks for the answer 55, otherwise no marks.

17

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| 5. | (a) | 5, 7, 9, 11 or $5 + 2 + 2 + 2 = 11$ or $5 + 6 = 11$ use $a = 5$, $d = 2$, $n = 4$ and $t4 = 5 + 3 \times 2 = 11$ B1 1 | |
|----|-----|--|---|
| | | B1 Any other sum must have a convincing argument | |
| | (b) | $t_n = a + (n - 1)d \text{ with one of } a = 5 \text{ or } d = 2 \text{ correct}$ (can have a letter for the other) = 5 + 2(n - 1) \text{ or } 2n + 3 \text{ or } 1 + 2(n + 1) A1 2 | 2 |
| | | M1 for an attempt to use $a + (n - 1)d$ with one of <i>a</i> or <i>d</i> correct (the other can be a letter) Allow any answer of the form $2n + p$ ($p \neq 5$) to score M1. | |
| | | A1 for a correct expression (needn't be simplified) [Beware $5 + (2n - 1)$ scores A0] Expression must be in <i>n</i> not <i>x</i> . Correct answers with no working scores 2/2. | |
| | (c) | $S_n = \frac{n}{2} [2 \times 5 + 2(n-1)]$ or use of $\frac{n}{2} (5 + \text{``their } 2n + 3\text{''})$ | |
| | | (may also be scored in (b)) = $\{n(5+n-1)\} = n(n+4)$ (*) M1A1 A1cso 3 | 5 |
| | | M1 for an attempt to use S_n formula with $a = 5$ or $d = 2$ or $a = 5$ and their " $2n + 3$ " | |
| | | 1 st A1 for a fully correct expression | |
| | | 2^{nd} A1 for correctly simplifying to given answer. No incorrect working seen. Must see S_n used. Do not give credit for part (b) if the equivalent work is given in part (d) | |
| | (d) | 43 = 2n + 3 M1 [n] = 20 A1 2 | 2 |
| | | M1 for forming a suitable equation in n (ft their (b)) and attempting to solve leading to $n =$ | |
| | | A1 for 20 Correct answer only scores 2/2. Allow 20 following a restart but check working. eg $43 = 2n + 5$ that leads to $40 = 2n$ and $n = 20$ should score M1A0. | |
| | (e) | $S_{20} = 20 \times 24, = \underline{480} \text{ (km)}$ M1A1 2 | 2 |
| | | M1 for using their answer for n in $n(n + 4)$ or S_n formula, their n must be a value. | |
| | | A1 for 480 (ignore units but accept 480 000 m etc) [no matter where their 20 comes from] | |

2

NB "attempting to solve" eg part (d) means we will allow sign slips and slips in arithmetic but not in processes. So dividing when they should subtract etc would lead to M0. Listing in parts (d) and (e) can score 2 (if correct) or 0 otherwise in each part. Poor labelling may occur (especially in (b) and (c)).

If you see work to get n(n + 4) mark as (c)

[10]

6

(a)

= -6

$$u_{25} = a + 24d = 30 + 24 \times (-1.5)$$
 M1
= -6 A1

- M: Substitution of a = 30 and $d = \pm 1.5$ into (a + 24d). Use of a + 25d (or any other variations on 24) scores M0.
- a + (n-1)d = 30 1.5(r-1) = 0M1 (b) r = 21A1 2
 - M: Attempting to use the term formula, equated to 0, to form an equation in r (with no other unknowns). Allow this to be called *n* instead of *r*. Here, being 'one off (e.g. equivalent to a + nd), scores M1.

(c)
$$S_{20} = \frac{20}{2} \{60 + 19(-1.5)\} \text{ or } S_{21} = \frac{21}{2} \{60 + 20(-1.5)\}$$

or $S_{21} = \frac{21}{2} \{30 + 0\}$
= 315 M1A1ft
A1 3

- M: Attempting to use the correct sum formula to obtain S20, S21, or, with their r from part (b), S_{r-1} or S_r .
- 1^{st} A(ft): A correct numerical expression for S20, S21, or, with their r from part (b), S_{r-1} or S_r ... but the ft is dependent on an integer value of r.

Methods such as calculus to find a maximum only begin to score marks after establishing a value of *r* at which the maximum sum occurs. This value of r can be used for the M1 A1ft, but must be a positive integer to score A marks, so evaluation with, say, n = 20.5 would score M1 A0 A0.

'Listing' and other methods

- M: Listing terms (found by a correct method), and picking the 25^{th} term. (a) (There may be numerical slips).
- (b) M: Listing terms (found by a correct method), until the zero term is seen. (There may be numerical slips). 'Trial and error' approaches (or where working is unclear or non-existent) score M1 A1 for 21, M1 A0 for 20 or 22, and M0 A0 otherwise.
- M: Listing sums, or listing and adding terms (found by a correct method), (c) at least as far as the 20th term. (There may be numerical slips). A2 (scored as A1 A1) for 315 (clearly selected as the answer). 'Trial and error' approaches essentially follow the main scheme, beginning to score marks when trying S_{20} , S_{21} , or, with their r from part (b), S_{r-1} or S_r .

If no working (or no legitimate working) is seen, but the answer 315 is given, allow one mark (scored as M1 A0 A0).

For reference:

Sums: 30, 58.5, 85.5, 111, 135, 157.5, 178.5, 198, 216, 232.5, 247.5, 261, 273, 283.5, 292.5, 300, 306, 310.5, 313.5, 315,

| 7 | (a) | | tify $a = 5$ and $d = 2$ $a_0 = a + (200 - 1)d$ (= 5 + (200 - 1)) | May be implied ~ 2 | B1 M1 | |
|---|-----|-----|---|-----------------------------|----------|---|
| | | · - | $\underline{0}(-)a + (200 - 1)a + (-5 + (200 - 1))a$ $\underline{0}(p) \text{ or } (\underline{t})\underline{4.30}$ | ~ 2) | A1 | 3 |
| | | B1 | can be implied if the correct answer If 403 is <u>not</u> obtained then the value be clearly identified as $a = 5$ and $d =$ This mark can be awarded at any | es of a and d must = 2. | | |
| | | M1 | for attempt to use <i>n</i> th term formula Follow through their <i>a</i> and <i>d</i> . Must have use of $n = 200$ and one correct follow through. Must be 199 not 200. | | | |
| | | A1 | for 403 or 4.03 (i.e. condone missin Condone £403 here. | g £ sign here). | | |
| | | NB | a = 3, $d = 2$ is B0 and $a + 200d$ is M and A1 if it leads to 403. Answer only of 403 (or 4.03)scores | | 31M1 | |
| | | ALT | Listing | | | |
| | | | They might score B1 if $a = 5$ and d Then award M1A1 together for 403 | • | 1. | |

[7]

3

(b)
$$(S_{200} =) \frac{200}{2} [2a + (200 - 1)d]$$
 or $\frac{200}{2} (a + \text{``their 403''})$ M1

$$= \frac{200}{2} [2 \times 5 + (200 - 1) \times 2] \text{ or } \frac{200}{2} (5 + \text{``their 403''})$$
A1
= 40 800 or £408 A1

$$=$$
 40 800 or £408

for use of correct sum formula with n = 200. M1 Follow through their *a* and *d* and their 403. Must have <u>some</u> use of n = 200, and some of a, d or lcorrect follow through.

1stA1 for any correct expression (i.e. must have
$$a = 5$$
 and $d = 2$)
but can f.t. their 403 still.

2ndA1 for 40800 or £408 (i.e. £ sign is required before we accept 408 this time). 40800p is fine for A1 but £40800 is A0.

ALT Listing

$$\sum_{r=1}^{200} (2r+3)$$
. Give M1 for $2 \times \frac{200}{2} \times (201) + 3k$ (with $k > 1$),
A1 for $k = 200$ and A1 for 40800.

M1: Attempted use of term formula for arithmetic series, or... answer in the form (3n + constant), where the constant is a non-zero value

Answer for (a) does not require simplification, and a correct answer without working scores all 3 marks.

(b)
$$S_n = \frac{n}{2} \{ 2a + (n-1)d \} = \frac{10}{2} \{ 8 + (10-1) \times 3 \}, = 175,$$
 M1A1, A1 3

- M1: Use of correct sum formula with n = 9, 10 or 11.
- A1: Correct, perhaps unsimplified, numerical version. A1: 175

Alternative: (Listing and summing terms).

- M1: Summing 9, 10 or 11 terms. (At least 1st, 2nd and last terms must be seen).
- A1: Correct terms (perhaps implied by last term 31). A1: 175

[6]

Alternative: (Listing all sums)

- M1: Listing 9, 10 or 11 sums. (At least 4, 7,, "last").
- A1: Correct sums, correct finishing value 175. A1: 175

Alternative: (Using last term).

M1: Using
$$S_n = \frac{n}{2}(a+l)$$
 with T9, T10 or T11 as the last term.

A1: Correct numerical version $\frac{10}{2}(4+31)$. A1: 175

Correct answer with <u>no</u> working scores 1 mark: 1,0,0.

(c)
$$S_k < 1750: \frac{k}{2} \{8+3(k-1)\} < 1750 \left(\text{ or } S_{k+1} > 1750: \frac{k+1}{2} \{8+3k\} > 1750 \right) M1$$

 $3k^2 + 5k - 3500 < 0 \text{ (or } 3k^2 + 11k - 3492 > 0)$ M1A1
(Allow equivalent 3-term versions such as $3k^2 + 5k = 3500$).
 $(3k - 100)(k + 35) < 0$
Requires use of correct inequality throughout.(*) A1cso 4
For the first 3 marks, allow any inequality sign, or equals.

 1^{st} M: Use of correct sum formula to form inequality or equation in *k*, with the 1750.

 2^{nd} M:(Dependent on 1^{st} M). Form 3-term quadratic in *k*.

1st A: Correct 3 terms.

Allow credit for part (c) if valid work is seen in part (d).

(d)
$$\frac{100}{3}$$
 or equiv. seen $\left(\text{or } \frac{97}{3} \right)$, $k = 33$ (and no other values) M1, A1 2

Allow both marks for k = 33 seen without working. Working for part (d) must be seen in part (d), not part (c).

[12]

A1A1

9
$$a + (n-1)d = k$$
 $k = 9 \text{ or } 11$ M1
 $(u_{11} =) a + 10d = 9$ A1c.a.o.

$$\frac{n}{2} [2a + (n-1)d] = 77$$

or $\frac{(a+1)}{2} \times n = 77$
 $l = 9 \text{ or } 11$ M1

$$(S_{11} =) \frac{11}{2} (2a+10d) = 77 \text{ or } \frac{(a+9)}{2} \times 11 = 77$$
 A1

eg
$$a + 10d = 9$$
 or $a + 9 = 14$
 $a + 5d = 7$ M1

a = 5 and d = 0.4 or exact equivalent

 I^{st} M1 Use of u_n to form a linear equation in a and d. a + nd = 9 is M0A0 I^{st} A1 For a + 10d = 9.

 $2^{nd} M1$ Use of S_n to form an equation for a and d (LHS) or in a (RHS)

 2^{nd} A1A correct equation based on S_n .

For 1^{st} 2 Ms they must write n or use n = 11.

 3^{rd} M1 Solving (LHS simultaneously) or (RHS a linear equation in a) Must lead to a = ... or d = ... and depends on one previous M

$$3^{rd}$$
 A1 for $a = 5$

 $4^{th} A1$ for d = 0.4 (o.e.)

 $\underline{ALT} \quad Uses \ \frac{(a+l)}{2} \times n = 77 \ to \ get \ a = 5, \ gets \ second \ and \\ third \ M1 \ A1 \ i.e. \ 4/7 \\ Then \ uses \ \frac{n}{2} [2a+(n-1)d] = 77 \ to \ get \ d, \ gets \ 1^{St} \ M1 \\ A1 \ and \ 4^{th} \ A1 \\ \underline{MR} \quad Consistent \ MR \ of \ 11 \ for \ 9 \ leading \ to \ a = 3, \ d = 0.8 \\ scores \ M1A0M1A0M1 \ A1ftA1ft \\ \end{bmatrix}$

[7]

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| 10. | (a) | a + (a + d) = £ (500 + 500 + 200) = £1200 | cso | B1 | 1 | |
|-----|-----|--|-----|---------------|---|------|
| | (b) | $a = 500, d = 200;$ $u_{\rm S} = a + (8 - 1)d$ | | M1 | | |
| | | $= \pounds(500 + 7 \times 200) = \pounds1900$ | | A1 | 2 | |
| | (c) | $S_8 = \frac{8}{2} \ (2 \times 500 + (8 - 1) \times 200)$ | Ν | 11 A1 | | |
| | | $= \pm 9600$ | | A1 | 3 | |
| | (d) | $\frac{n}{2}(1000 + (n-1)\ 200) = 32\ 000$ | Ν | 1 1 A1 | | |
| | | $n^{2} + 4n - 320 = 0$ M1 reducing to a 3 term quadratic A1 any multiple of the above | Ν | 11 A1 | | |
| | | (n+20)(n-16) = 0 | | M1 | | |
| | | n = 16 | | A1 | | |
| | | Age is 26 | | A1 | 7 | |
| | | | | | | [13] |

In (b) if the sum is found by repeated addition, i.e.

$$u^{1} = \text{\pounds}500, u_{2} = \text{\pounds}700, u_{3} = \text{\pounds}900, u_{4} = \text{\pounds}1100, u_{5} = \text{\pounds}1300,$$

 $u_{6} = \text{\pounds}1500, u_{7} = \text{\pounds}1700, u_{8} = \text{\pounds}1900,$

allow M1 A1 at completion.

If for (c) these 8 terms are added up, allow M1 A2 at completion. Do notdivide the As with this method if (b) has been completed similarly. If only(c) is done by repeated addition allow A1 if the individual terms are correct if a complete method is shown.

| 11. | (a) | $S = a + (a + d) + (a + 2d) + \dots + [a + (n - 1)d]$ | B1 | |
|-----|-----|---|-------|---|
| | | $S = [a + (n-1)d] + [a + (n-2)d] + \dots + a$ or equiv. | M1 | |
| | | Add: $2S = n[2a + (n-1)d] \Longrightarrow S = n[2a + (n-1)d] \operatorname{cso}(*)$ | M1 A1 | 4 |
| | | B1: requires min of 3 terms, including the last. | | |

| (b) | 3, 8, 13 | | B1 | 1 |
|-----|----------|---|----|---|
| | | First M1 generous; second M1 hard. | | |
| | | Note: Result is given so check working carefully. | | |

(c)
$$a = 3$$
 $d = 5$
 $Sum = \frac{1}{2}n[(2 \times 3) + 5(n - 1)] = \frac{1}{2}n(5n + 1)$ (*) B1ft
M1 A1 3

For B1 f.t. 3 terms must be in AP, But allow M1 for candidate's "a" and "d" in given result in (a)

EXTRA

$$5\sum_{r} r - \sum_{r} 2$$
 B1

$$=5\frac{n(n+1)}{2}-2n$$
 M1

$$=\frac{5n^2+n}{2}=\frac{n(5n+1)}{2} (*) (\cos)$$
A1

(d) Finding
$$\sum_{1}^{200}$$
 e.g. $\sum_{r=1}^{200} (5r-2) = \frac{1}{2} \times 200 \times 1001$ (= 100100) M1

Sum of first 4 terms:
$$\sum_{r=1}^{4} (5r-2) = \frac{1}{2} \times 4 \times 21$$
 or 42 stated B1
 $\sum_{r=5}^{200} (5r-2) = S(200) - S(4) = 100100 - 42 = 100058$ M1A1 4

ALT: Working with 23, 28, 33,

$$a = 23$$
 B1; Finding "n" and d M1
Applying $S = \frac{1}{2}n[2a + (n - 1)d]$ with candidate's 23, $n = 195$ or 196,
 $d = 5$ M1
First M1 for substitution of 200 in result from (c)

S.C. Allow second M1 for S(200) - S(5)

[12]

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| 12. | (a) | $(S =)a + (a + d) + \dots + [a + (n - 1)d]$ B1 $(S =)[a + (n - 1)d] + \dots + a$ M1 $2S = [2a + (n - 1)d] + \dots + [2a + (n - 1)d]$ either dM1 2S = n[2a + (n - 1)d] |
|-----|-----|--|
| | | $S = \frac{n}{2} [2a + (n-1)d]$ A1 c.s.o. 4 |
| | | <i>B1 requires at least 3 terms, must include first and last terms, an adjacent term and dots + signs.</i> |
| | | I st M1 for reversing series. Must be arithmetic with a, n and d or l. (+ signs not essential here) |
| | | 2 nd dM1 for adding, must have 2S and be a genuine attempt. Either line is sufficient. |
| | | Dependent on 1 st M1 |
| | | (NB Allow first 3 marks for use of l for last term but as given for final mark) |
| | (b) | (a = 149, d = -2) $u_{21} = 149 + 20(-2) = \pounds 109$ M1 A1 2 |
| | | M1 for using $a = 149$ and $d = \pm 2$ in $a + (n - 1)d$ formula. |
| | (c) | $S_n = \frac{n}{2} [2 \times 149 + (n-1)(-2)] (n(150 - n)) $ M1 A1 |
| | | $S_n = 5000 \Rightarrow n^2 - 150n + 5000 = 0$ (*) A1 c.s.o. 3 |
| | | M1 for using their a , d in $S_n A1$ any correct expression |
| | | A1cso for putting $S_n = 5000$ and simplifying to given expression. No wrong work |
| | (d) | (n-100)(n-50) = 0 M1 n = 50 or 100 A2/1/0 3 |
| | | M1 Attempt to solve leading to $n =$ A2/1/0 Give A1A0 for 1 correct value and A1A1 for both correct |
| | (e) | $u_{100} < 0 \therefore n = 100$ not sensible B1 ft 1 |
| | | B1 f.t. Must mention 100 and state u100 < 0 (or loan paid or equivalent) |
| | | If giving f.t. then must have $n \ge 76$. |
| | | |
| | | |
| | | |
| | | |

[13]

13. (a) -3, -1, 1
 B1 B1
 2

 B1: One correct
 B1 B1
 2

 (b) 2
 B1 ft
 1

 (ft only if terms in (a) are in arithmetic progression)
 B1 ft
 1

 (c) Sum =
$$\frac{1}{2}n\{2(-3) + (n-1)(2)\}$$
 or $\frac{1}{2}n\{(-3) + (2n-5)\}$
 M1 A1 ft

 $= \frac{1}{2}n\{2n-8\} = n(n-4)$
 (Not just $n^2 - 4n$)
 (*)
 A1
 3

14. (a)
$$(5p-8) - p = (3p+8) - (5p-8)$$
M1Solve, showing steps, to get $p = 4$, or verify that $p = 4$. (*)A1 c.s.o.2Alternative:Using $p = 4$, finding terms (4, 12, 20), and indicating
differences.[M1]Equal differences + conclusion (or "common difference = 8").[A1]

(b)
$$a = 4$$
 and $d = 8$ (stated or implied here or elsewhere). B1
 $T_{40} = a + (n-1)d = 4 + (39 \times 8) = 316$ M1 A1 3

(c)
$$S_n = \frac{1}{2}n[2a + (n-1)d] = \frac{1}{2}n[8 + 8(n-1)]$$
 M1 A1ft
= $4n^2 = (2n)^2$ A1 3

(b)
$$d = 74 - 77 = -3$$
 B1 ft 1

(c)
$$S_{50} = \frac{1}{2}n[2a+9n-1)d] = 25[(2 \times 77) + (49 \times -3)]$$
 M1 A1 ft
= 175 A1 3

Alternative method: Find last term, then use $\frac{1}{2}n(a+1)$

C1 Sequences and series: Arithmetic series - Mark Schemes

[6]

[8]

[6]

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16. (a) (i)
$$a + (n-1)d = 280 + (35 \times 5) = 455$$
 M1 A1 2
(ii) $\frac{1}{2}n [2a + (n-1)d] = 18 [560 + (35 \times 5)] = 13 230$ M1 A1 ft 2
(b) $18 [560 + (35 \times d)] = 17 000$ M1 A1
 $d = 10.98...$ $x = 11$ (allow 11.0 or 10.98 or 10.99 or $10\frac{62}{32}$)M1 A1 4

[8]

17. (a)
$$S = a + (a + d) + (a + 2d) + ... + a + (n - 1)d$$
 B1
 $S = a + (n - 1)d + a + (n - 2)d + ... + a$ M1
Adding,

$$2S = n[2a + (n-1)d] \Rightarrow S = \frac{n}{2}[2a + (n-1)d]$$
 M1 A1 4

(b)
$$k+k+7=2 \times 7.5 \Rightarrow k=4$$
 M1 A1 2

(c)
$$S_{31} = \frac{31}{2} [2 (4) + 30 (3 \frac{1}{2})]$$
 M1 A1 ft
1751.5 M1 A1 4

[10]

18. (a)
$$S = a + (a + d) + \dots + [a + (n - 1)d]$$
 B1
 $S = [a + (n - 1)d] + \dots + a$ M1
Add: $2S = n[2a + (n - 1)d], S = \frac{1}{2}n[2a + (n - 1)d]$ (*) M1 A1 4
(b) $a + 15d = 6$ B1
 $\frac{1}{2}n[2a + (n - 1)d] = 8(2a + 15d) = 72$ M1 A1
Solve simultaneously: $a = 3$ 3cm M1 A1 5
(c) $a = 3$: $15d = 6 - 3 = 3$ $d = 0.2$ M1 A1 2
[11]

1. Part (a) was often answered correctly but some quoted a + 29d but failed to use the value of 40.75 to form an equation. Most scored well in part (b) but some failed to give sufficient working to earn both marks in this "show that" question. A successful solution requires the candidates to show us clearly their starting point (which formula they are using) and then the values of any variables in this formula. Those using

 $\frac{n}{2}(a+l)$ in particular needed to make it clear what value of *n* they were using.

Candidates might also consider that a two mark question will usually require 2 steps of working to secure the marks.

Many students had a correct strategy for finding a and d but not always a sensible strategy for doing so without a calculator. Starting from the given equation in part (b) the "sensible" approach is to divide both sides by 15 and then subtract 40.75 even this

though proved challenging for some with errors such as $\frac{1005}{15} = 61$ and

67 - 40.75 = 27.25 spoiling a promising solution. Those who chose the more difficult expansion of the bracket in part (b) often got lost in the ensuing arithmetic. A number of candidates failed to spot that $\frac{14.5}{29} = \frac{1}{2}$ and lost the final mark.

It was encouraging though to see most candidates using the given formulae to try and solve this problem; there were very few attempting a trial and improvement or listing approach.

- 2. Most candidates interpreted the context of this question very well and it was common for full marks to be scored by those who were sufficiently competent in arithmetic series methods. Answers to parts (a) and (b) were usually correct, with most candidates opting to use the appropriate formulae and just a few resorting to writing out lists of numbers. In part (c), it was pleasing that many candidates were able to form a correct equation in A. Disappointing, however, were the common arithmetical mistakes such as $4100 \div 20 = 25$. Trial and improvement methods in part (c) were occasionally seen, but were almost always incomplete or incorrect.
- 3. Many candidates did not seem to realise that this question was about Arithmetic Series initially and started with a simple arithmetic approach to obtain a difference of 60. When it came to part (b) they often did start to use arithmetic series formulae but confusion over the value of *n* and the sign of their 60 meant that an incorrect value for *a* was common. In part (c) most realised that the formula for *S*40 was required and their values were often substituted correctly.

4. Although most candidates made a reasonable attempt at this question, only those who demonstrated good skills in algebra managed to score full marks. The structure of parts (a) and (b) was intended to help candidates, but when the initial strategy was to write down (correctly) 3d = 32.5 - 25, there was sometimes confusion over what was required for the two equations in part (a). Even when correct formulae such as $u_{18} = a + 17d$ were written down, the substitution of $u_{18} = 25$ did not always follow. The work seen in these first two parts was often poorly presented and confused, but credit was given for any valid method of obtaining the values of *d* and *a* without assuming the value of *a*. In part (c), many candidates managed to set up the correct sum equation but were subsequently let down by poor arithmetic or algebra, so were unable to proceed to the given quadratic equation. Being given 55×40 (to help with the factorisation in the last part of the question) rather than 2200 sometimes seemed to be a distraction.

Despite being given the 55×40 , many candidates insisted on using the quadratic formula in part (d). This led to the problem of having to find the square root of 9025 without a calculator, at which point most attempts were abandoned.

5. Most gave a convincing argument in part (a) but in part (b) some merely quoted the formula for the *n*th term and failed to substitute values for *a* and *d*. Many simplified their answer here to 2n + 3 and some gave the incorrect 2n + 5. Part (c) was difficult for some and even those who started with a correct expression could not always complete the simplification with $\binom{n}{2}(8 + 2n)$ being simplified to n(16 + 4n). Apart from the condition

simplification with $\frac{n}{2}(8+2n)$ being simplified to n(16+4n). Apart from the candidates who tried to solve $43 = S_n$ instead of u_n part (d) was usually answered correctly and often this was followed by a correct answer to part (e). A number of candidates though didn't appreciate that *n* had a value at this stage and they simply repeated their answer from part (c).

6. Most candidates knew in part (a) how to use the term formula for an arithmetic sequence.

Some, effectively using a + nd instead of a + (n - 1)d, reached the answer -7.5 instead of -6, while the omission of a minus sign was a surprisingly common mistake, leading to 30 + 36 = 66 instead of 30 - 36 = -6.

In part (b), many candidates equated the correct expression to zero to score the method mark, but mistakes in calculation were very common. Dividing 31.5 by 1.5 sometimes

caused problems. Other approaches, such as counting back from the 25th term found in part (a), were sometimes successful.

Few students seemed to fully appreciate the connection between part (b) and part (c) but those who did invariably scored all the marks. Many ended up trying to solve an equation with two unknowns (S_n and n) or assumed that S_n was zero, which led to the commonly seen, incorrect n = 41. Many candidates seemed completely confused by part (c) and made no real progress. In the question as a whole, inefficient methods involving 'listing' terms were infrequently seen.

7. This question was answered well with most candidates quoting and using the appropriate arithmetic series formulae. In part (a) the majority obtained 403 and some wrote their answer as £4.03, there was the usual crop of arithmetic errors with $2 \times 199 = 298$ or 498 being quite common. In part (b) the sum formula was usually quoted and used correctly but again arithmetic slips (e.g. $408 \times 100 = 4080$ or 10 + 398 = 418 or 498) were often seen and some made errors with the units giving the answer as £40800. There were some cases of candidates trying to use the $n \frac{(a+l)}{2}$ formula and misreading

the l for a 1.

8. There were many good attempts at this question, although few candidates scored full marks. The vast majority recognised that an arithmetic series was involved. In part (a), most candidates found a correct expression, either directly or by using the formula a +(n-1)d. A few, however, offered a recurrence relationship. There were occasional numerical slips in the evaluation of the sum in part (b), but many correct answers were seen. The majority of candidates used the sum formula rather than a list of terms. It was disappointing here to see frequent misunderstanding (or misreading) of the question leading to the answer 31, the tenth term rather than the sum of the first ten terms. Those who realised the need for the sum formula in part (c) usually made good progress, but a significant number simply started to expand and proceeded to solve the given equation, possibly producing work that was relevant only to part (d). Those who did this often wasted time trying to use the quadratic formula on their expanded version of the equation. In the better attempts, the inequality was often introduced without justification at a late stage in the working, losing the final mark. Some candidates confused the sum (1750) with the first term of the series and made no progress.

In part (d), the majority of candidates found the value $\frac{100}{3}$ but did not continue to interpret this result in the context of the question. The final answer was often given as a fractional value, a negative value or a set of values of *k*.

9. There were many good responses to this question and candidates who used the correct formulae for arithmetic equations were often able to solve two simultaneous equations and reach the answers quickly. Some candidates showed weaknesses in algebraic processing e.g. 11(10 + 10d) leading to 110 + 10d or 10d = 4 followed by d = 10/4 = 2.5. There were the inevitable arithmetic errors too e.g. $2 \times 77 = 144$ was common. Careful simplification of equations at each stage would avoid the need for such difficult calculations and the subsequent risk of errors. Candidates should be encouraged to see that $\frac{11}{2}(2a + 10 d) = 77$ can be simplified to $\frac{1}{2}(2a + 10 d) = 7$ or even better a + 5d = 7.

Inevitably some candidates used a trial and improvement approach to this question, this is not recommended, wastes time and is particularly susceptible to errors. By contrast some candidates found very slick and efficient solutions to the question.

10. Pure Mathematics P1

The procedures needed for solving this question were well understood but candidates often got confused with the details of the description in the question. With the approach that most used for parts (b) and (c), the value of n needed for the standard formulae quoted was n = 8 but the values 7, 9, 17 and 18 were all used by a number of candidates. A few candidates confused parts (b) and (c) giving the answer to (c) in (b) and giving a further accumulation of numbers in part (c). Most knew how to set up an equation for solving part (d). The large numbers meant that errors were seen in "cancelling" zeros but many reached a correct solution n = 16. Here, however, difficulty was found in interpreting this answer and the incorrect "Alice is 27" was commoner than the correct age of 26. Some thought that she was 16, failing to see that this answer contradicted their correct answer to part (c). A few candidates gave a solution to this question involving no algebra and using only the procedures of arithmetic. As long as all the calculations were shown, this was accepted.

Core Mathematics C1

Many candidates found this question, an arithmetic series in context, difficult. It was very common to see multiple attempts at parts (b), (c) and especially (d). While most candidates used the arithmetic series formulae efficiently, a significant minority produced long lists of numbers and calculations, which sometimes led to correct answers but which were often poorly presented and difficult for examiners to interpret.

Part (a) was intended to help candidates understand what was happening, and nearly all gave a correct explanation, although some made heavy weather of this, perhaps using the sum formula for 2 terms.

In parts (b) and (c), just a few candidates confused term and sum, and a few thought that the common difference was £700 rather than £200, but apart from this the most common mistake was to use a wrong value of *n* such as n = 18 (the girl's age).

Pleasingly, candidates often managed to set up a quadratic equation in part (d), although the resulting algebra defeated many. An over-reliance on the quadratic formula (rather

than factorisation) gave a much more difficult task for this particular equation, $n^2 + 4n - 320 = 0$. Those who successfully reached n = 16 sometimes lost the last mark by not giving the girl's age or by giving it as 27 rather than 26.

11. Candidates who had learnt a proof for part (a) were often able to reproduce it, but such candidates were in the minority and full marks for this part were seen infrequently. Most candidates gained the mark in part (b), and part (c) was often a good source of marks. However, the modal mark in part (d) was one; the most common answers being100100 {just considering S(100)} and 104100 {considering an AP with first term 23 but with 200 terms}. Only the better candidates had a complete strategy, correct solutions being equally divided between finding S(100) – S(4), although S(100) – S(5) was common, and using a formula for the sum of an AP with first term 23 and 196 terms.

- 12. Whilst it was clear that many centres had encouraged their students to learn an appropriate proof for part (a) there were a surprising number of students who were not well prepared for this part of the question and the success rate was under 50%. Errors included a lack of + signs and use of a + nd. Some candidates attempted to add the terms in pairs, but gave no consideration to series with an odd number of terms. In part (b) the answer was usually correct although sum tried to use d = 2 and others used the formula for the sum of 21 terms. Part (c) was generally answered quite well too although there was some inappropriate altering of signs to achieve the printed result. Those who factorized in part (d) had no problems here but a disappointing number attempted to use the formula and were defeated by the arithmetic. In part (e) a common incorrect answer was that 100 months (or years) was too long rather than stating that the loan would have been repaid by then or that the 100^{th} term was negative.
- 13. Although just a few candidates were completely confused by the demands of this question, the vast majority scored full marks in parts (a) and (b). An occasional mistake in part (b) was to give -2 rather than 2 as the common difference. Part (c), however, in which a general formula for the sum to *n* terms had to be established, caused more difficulties. Those who used the arithmetic series sum formula with *n* as the number of terms were usually able to proceed convincingly to the given answer, but no credit was given to candidates who simply verified the given formula for a few specific values of *n*.
- 14. Candidates sometimes made hard work of part (a), bringing a and d as well as p into their equations, but the majority were able to show convincingly that p was equal to 4. A few confused this with the geometric series and began by equating ratios of terms rather than differences. In part (b), the formula for the *n*th term of an arithmetic series was usually quoted correctly, and there were many correct answers for the 40^{th} term. Some candidates found the sum of the first forty terms instead of the 40^{th} term.

There was, however, less success in part (c), where candidates were sometimes confused as to what was required. Some launched into the general proof for the sum formula, while others simply verified the result for particular values of n. Those who did proceed correctly to $4n^2$ often failed to explain why this was a perfect square, losing the final mark.

15. Most candidates understood the S notation and were able to write down the first two terms correctly in part (a), although 80 was occasionally thought to be the first term. In part (b), the answer 3 for the common difference was as popular as the correct answer – 3. Knowledge of the sum formulae for the arithmetic series was generally good in part (c), where few candidates quoted geometric series formulae.

- 16. Most candidates recognised this question as an application of the arithmetic (rather than geometric) series, and those who knew the correct formulae often produced excellent solutions to all three parts. In part (a), there was occasional confusion between the term formula and the sum formula, but candidates usually corrected themselves, sometimes by re-labelling parts (i) and (ii). Those who did not know the formulae sometimes resorted to lengthy calculations and lost marks if they did not show clearly how they had found their answers. Using (a + 36d) instead of (a + 35d) was a common mistake in part (i), and unless the correct formula was seen, this lost both marks. Candidates using a wrong formula in part (a)(ii) and the same wrong formula in part (b) were given some credit for method in (b), but this sometimes led to answers which should have been recognised as clearly absurd. Some candidates were successful with a "trial and improvement" approach in (b), but it should be noted that in this case examiners needed to see sufficient evidence of trials giving totals above and below 17000, with calculation of corresponding values.
- **17.** No Report available for this question.
- **18.** No Report available for this question.