Core 1 Polynomials Questions

6 The polynomial $p(x)$ is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Hence express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

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6 The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, r, when $p(x)$ is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express $p(x)$ in the form

$$(x - 2)(x^2 + ax + b) + r$$

where a, b and r are constants. *(4 marks)*

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1 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of $p(x)$, show that $k = 10$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

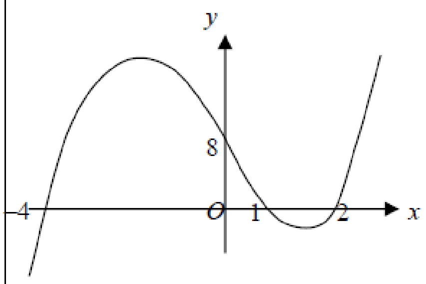
6 (a) The polynomial $\mathrm{f}(x)$ is given by $\mathrm{f}(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

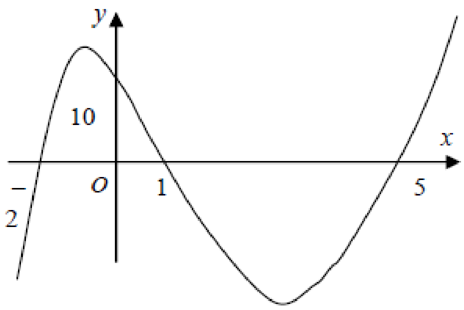
(ii) Express $\mathrm{f}(x)$ in the form $(x - 1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $\mathrm{f}(x) = 0$ has exactly one real root and state its value. *(3 marks)*

Core 1 Polynomials Answers

6(a)(i)	$\mathrm{p}(2) = 8 + 4 - 20 + 8$	M1		Finding p(2) M0 *long division*
	$= 0$, $\Rightarrow$ $x - 2$ is a factor	A1	2	Shown = 0 **AND** conclusion/ statement about $x - 2$ being a factor
(ii)	Attempt at quadratic factor	M1		or factor theorem again for 2nd factor
	$x^2 + 3x - 4$	A1		or $(x+4)$ or $(x-1)$ proved to be a factor
	$\mathrm{p}(x) = (x-2)(x+4)(x-1)$	A1	3	
(b)		B1		Graph through (0,8) 8 marked
		B1√		Ft "their factors" 3 roots marked on x-axis
		M1		Cubic curve through their 3 points
		A1	4	Correct including x- intercepts correct Condone max on y-axis etc or slightly wrong concavity at ends of graph
	Total		**9**	

6(a)	$\mathrm{p}(3) = 27 - 36 + 9$	M1		Finding p(3) and **not** long division
	$\mathrm{p}(3) = 0 \Rightarrow x - 3$ is a factor	A1	2	Shown = 0 **plus a statement**
(b)	$x(x^2 - 4x + 3)$ or $(x-3)(x^2 - x)$ attempt	M1		Or $\mathrm{p}(1) = 0 \Rightarrow x - 1$ is a factor attempt
	$\mathrm{p}(x) = x(x-1)(x-3)$	A1	2	Condone $x + 0$ or $x - 0$ as factor
(c)(i)	$\mathrm{p}(2) = 8 - 16 + 6$	M1		Must use p(2) and **not** long division
	(Remainder is) -2	A1	2	
(ii)	Attempt to multiply out and compare	M1		Or long division (2 terms of quotient)
	coefficients $a = -2$	A1		$x^2 - 2x\ldots$
	$b = -1$	A1		-1
	$r = -2$	A1	4	Withhold final A1 for long division unless
	SC B1 for r = –2 if M0 scored			written as $(x-2)(x^2 - 2x - 1) - 2$
	Total		**10**	

1(a)(i)	$p(-2) = -8 - 16 + 14 + k$ $p(-2) = 0 \Rightarrow -10 + k = 0 \quad \Rightarrow k = 10$ Must have statement if k=10 substitute	M1 A1	2	or long division or $(x+2)(x^2 - 6x + 5)$ **AG** likely withhold if $p(-2) = 0$ not seen
(ii)	$p(x) = (x+2)(x^2 + ... \quad 5)$ $p(x) = (x+2)(x^2 - 6x + 5)$ $\Rightarrow p(x) = (x+2)(x-1)(x-5)$	M1 A1 A1	3	Attempt at quadratic or second linear factor $(x-1)$ or $(x-5)$ <u>from factor theorem</u> Must be written as product
(b)	$p(3) = 27 - 36 - 21 + k$ (Remainder) $= k - 30 = \underline{-20}$	M1 A1	2	long division scores M0 Condone $k - 30$
(c)		B1 B1√ M1 A1	4	Curve thro' 10 marked on y-axis **FT** their 3 roots marked on x-axis Cubic shape with a max and min Correct graph (roughly as on left) going beyond -2 and 5 (condone max anywhere between $x = -2$ and 1 and min between 1 and 5)
	Total		**11**	

6(a)(i)	$f(1) = 1 + 4 - 5$ $\Rightarrow f(1) = 0 \Rightarrow (x-1)$ is factor	M1 A1	2	must find $f(1)$ NOT long division shown $= 0$ plus a statement
(ii)	Attempt at $x^2 + x + 5$ $f(x) = (x-1)(x^2 + x + 5)$	M1 A1	2	long division leading to $x^2 \pm x + ...$ or equating coefficients $p = 1, \ q = 5$ by inspection scores B1, B1
(iii)	$(x =)\ 1$ is real root Consider $b^2 - 4ac$ for their $x^2 + x + 5$ $b^2 - 4ac = 1^2 - 4 \times 5 = -19 < 0$ Hence no real roots (or only real root is 1)	B1 M1 A1	3	 not the cubic! CSO; all values correct plus a statement