Core 1 Integration Questions

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line $L$.

![Graph showing the curve and line L](image)

The points $A$ and $B$ have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the $x$-axis at the origin $O$ and crosses the $x$-axis at the point $(3, 0)$. The line $L$ cuts the curve at the point $D$ where $x = -1$ and touches the curve at $C$ where $x = 2$.

(a) Find the area of the rectangle $ABCD$.  

(b)  
(i) Find $\int (3x^2 - x^3) \, dx$.  

(ii) Hence find the area of the shaded region bounded by the curve and the line $L$. 

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\frac{dy}{dx}$;  

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$;  

(iii) show that $y$ is decreasing when $x^2 - 2x > 0$. 

(d) Solve the inequality $x^2 - 2x > 0$.  

(2 marks)

(3 marks)

(4 marks)

(2 marks)

(3 marks)

(2 marks)

(2 marks)
5 The curve with equation \( y = x^3 - 10x^2 + 28x \) is sketched below.

The curve crosses the \( x \)-axis at the origin \( O \) and the point \( A(3, 21) \) lies on the curve.

(b) (i) Find \( \int (x^3 - 10x^2 + 28x) \, dx \). \( \text{(3 marks)} \)

(ii) Hence show that \( \int_0^3 (x^3 - 10x^2 + 28x) \, dx = 56 \frac{1}{4} \). \( \text{(2 marks)} \)

(iii) Hence determine the area of the shaded region bounded by the curve and the line \( OA \). \( \text{(3 marks)} \)

6 The curve with equation \( y = 3x^5 + 2x + 5 \) is sketched below.

The curve cuts the \( x \)-axis at the point \( A(-1, 0) \) and cuts the \( y \)-axis at the point \( B \).

(a) (i) State the coordinates of the point \( B \) and hence find the area of the triangle \( AOB \), where \( O \) is the origin. \( \text{(3 marks)} \)

(ii) Find \( \int (3x^5 + 2x + 5) \, dx \). \( \text{(3 marks)} \)
(iii) Hence find the area of the shaded region bounded by the curve and the line $AB$.  

(4 marks)

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$.  

(3 marks)

(ii) Hence find an equation of the tangent to the curve at the point $A$.  

(1 mark)

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.

The curve cuts the $x$-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5) \, dx$.  

(3 marks)

(ii) Hence find the area of the shaded region bounded by the curve and the line $AB$.  

(4 marks)
Core 1 Integration Answers

### Question 8(a)
- **y_D = 3 + 1 = 4** or **y_C = 12 - 8 = 4**
- **Area ABCD = 3 × 4 = 12**

### Question 8(b)(i)
- \( x^3 - \frac{x^4}{4} \) \( + C \)
- **M1** or **A1 2**
  - Attempt at either y coordinate

### Question 8(b)(ii)
- **Sub limits -1 and 2 into their (b)(i) ans**
- \[ 8 - 4 \left( -\frac{1}{4} \right) = 5 \frac{1}{4} \]
- **Shaded area = “their” (rectangle– integral)**
- \( = 12 - 5 \frac{1}{4} = 6 \frac{3}{4} \)

### Question 8(c)(i)
- \( \frac{dy}{dx} = 6x - 3x^2 \)

#### Question 8(c)(ii)
- When \( x = 1, \ y = 2 \) when \( x = 1, \)
- \( \frac{dy}{dx} = 3 \) as ‘their’ grad of tgt
- **M1\( ^\wedge \)**
- **A1 3**
  - Any correct form \( y = 3x - 1 \) etc

### Question 8(d)
- **Decreasing when** \( \frac{dy}{dx} = 6x - 3x^2 < 0 \)
- \( 3(2x - x^2) < 0 \) \( \Rightarrow x^2 - 2x > 0 \)

### Question 8(b)(i)
- \( \frac{x^4}{4} - \frac{10x^3}{3} + 14x^2 \) \( + C \)

#### Question 8(b)(ii)
- \[ \left[ \frac{81}{4} - 90 + 126 \right] = 56 \frac{1}{4} \]

#### Question 8(b)(iii)
- **Area of triangle = 31 \( \frac{1}{2} \)**
- **Shaded Area = 56 \( \frac{1}{4} \) - triangle area**
- **A1 3**
  - Or equivalent such as \( 99 \frac{3}{4} \)

### Total
- **18**
6(a)(i) \[ B (0, 5) \]
Area \( AOB = \frac{1}{2} \times 1 \times 5 \]
\[ = 2\frac{1}{2} \] \( \quad \text{M1} \]
\( \text{A1} \) 3
Condone slip in number or a minus sign

(ii) \[ \frac{3x^6}{6} + \frac{2x^2}{2} + 5x \]
\( \text{or} \quad \frac{x^6}{2} + x^2 + 5x \)
\( \quad \text{(may have } c \text{ or not)} \) \( \quad \text{M1} \]
\( \text{A1} \) 3
One term correct
All correct unsimplified

(iii) Area under curve = \[ \int_{-3}^{0} f(x) \, dx \]
\[ [0] - \left[ \frac{1}{2} + 1 - 5 \right] \]
Area under curve = \( 3\frac{1}{2} \) \( \quad \text{M1} \]
\( \text{A1} \)
Correctly written or \( F(0) - F(-1) \) correct
Attempt to sub limit(s) of \(-1\) (and 0)
Must have integrated
CSO (no fudging)

Area of shaded region = \( 3\frac{1}{2} - 2\frac{1}{2} = 1 \) \( \quad \text{B1} \)
\( \text{FT} \) their integral and triangle (very generous)

(b)(i) \[ \frac{dy}{dx} = 15x^4 + 2 \]
when \( x = -1 \), gradient = 17
One term correct
All correct (no +c etc)
\( \text{A1} \) 3
cso

(ii) \( y = "\text{their gradient}"(x + 1) \) \( \quad \text{B1} \)
\( \text{FT} \)
Must be finding tangent - not normal any form e.g. \( y = 17x + 17 \)

Total 14

(b)(i) \[ \int \ldots dx = \frac{x^4}{4} + 2x^2 - 5x + c \] \( \quad \text{M1} \]
\( \text{A1} \) 3
One term correct unsimplified
Second term correct unsimplified
All correct unsimplified

(ii) \[ [4 + 8 - 10] - \left[ \frac{1}{4} + 2 - 5 \right] \]
\[ = 4\frac{3}{4} \] \( \quad \text{M1} \]
\( \text{A1} \)
correct use of limits 1 and 2;
\( F(2) - F(1) \) attempted

Area of \( \Delta = \frac{1}{2} \times 11 = 5\frac{1}{2} \)
\( \Rightarrow \) shaded area = \( 5\frac{1}{2} - 4\frac{3}{4} \)
\( = 3\frac{3}{4} \) \( \quad \text{B1} \)
correct unsimplified
combined integral of \( 7x - 6 - x^3 \) scores
M1 for limits correctly used then

A3 correct answer with all working correct